Issue Indivisibility as a Cause of Peace

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April 25, 2022

Abstract

By pessimistic accounts, issue indivisibility causes war by prohibiting actors from selecting mutually acceptable agreements. In contrast, when mixed with other bargaining frictions, this note shows that issue indivisibility can promote *peace*. With shifting power, moderate indivisibilities allow rising states to credibly promise concessions they otherwise could not, mitigating commitment problems. With asymmetric information, moderate indivisibilities can convince actors to issue safer demands. As a result, in both cases, the probability of war is non-monotonic in indivisibilities. The results therefore indicate that the theoretical relationship between indivisibilities and war is more ambiguous than the literature suggests.

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1 Introduction

Conventional wisdom indicates that indivisible issues—bargaining objects that must go to a single actor—are a force for war. Pessimistic accounts are unambiguous here. The logic is straightforward. Because war is costly, settlements exist in principle that leave both sides better off (Fearon 1995). However, the range of these mutually acceptable settlements might require dividing the good to some degree. Indivisibilities make that impossible, setting the stage for war (Iklé 1971; Hassner 2003; Toft 2006; Svensson 2007; Goddard 2009; Keels and Wiegand 2020). Indeed, researchers have pointed to indivisibilities as the root of situations as diverse as Northern Ireland's "Troubles", Indian-Pakistani fights over Kashmir, and the Israel-Palestinian conflict.

By less pessimistic accounts, issue indivisibility is a more neutral force. This line of thought sees side payments as a solution. Even with indivisibilities, remaining divisible goods inject liquidity into the system (Fearon 1995). As a result, indivisibilities only appear to cause problems when the value of the indivisible goods swamp the value of the divisible goods. For example, Pillar (2014, 24) writes "If the stakes are chiefly indivisible, so that neither side can get most of what it wants without depriving the other of most of what it wants, negotiations are less apt to be successful." Based on this, Walter (1999, 131) infers that such a problem "makes settlement less likely."

However, models of issue indivisibility have focused on proving the existence of the mechanism—i.e., why an indivisibility causes bargaining breakdown. The next important step is testing the generality of the relationship. Are indivisibilities always a force for war? I answer this question by pushing outward on two dimensions. First, I allow for full variation between complete indivisibility and complete divisibility. That is, I parameterize what portion of the good is non-negotiable and what portion is a divisible side payment. Second, I embed these indivisibilities into models with other mechanisms that cause war. Because the probability of war is non-zero with fully divisible objects, this opens up the possibility that indivisibilities will promote peace.

I show two distinct mechanisms where this arises. Under shifting power, moderate indivisibilities can prevent rising states from demanding as much post-shift. In other words, indivisibilities allow the rising state to make credible otherwise incredible commitments. This mitigates the mechanism that normally causes conflict. With the declining state seeing more value in post-shift periods, it opts against preventive war.

Under uncertainty, moderate indivisibility may prohibit the uninformed state from proposing its optimal demand under the risk-return tradeoff. This forces the state to choose a safer or riskier demand instead. I show that the safer option is sometimes optimal, resulting in a lower probability of war. In fact, indivisibilities can create environments where the only reasonable proposals guarantee peace.

These results help explain the mixed empirical results for the indivisibility mechanism (Mason, Weingarten Jr and Fett 1999; Tir 2003; Hensel and Mitchell 2005). According to the current understanding, "[b]argaining theories are hard pressed to explain [these] result[s], as more indivisible issues should produce *fewer* agreements" (Hensel and Mitchell 2005, 283). Perhaps as a result of the absence of compelling cases, the literature has been tempted to overlook indivisibilities as a cause of war. This paper flips the script. We must understand indivisibilities to understand why they cause peace.

2 Issue Indivisibilities as a Cause of War

I begin by building a basic bargaining model that permits full variation in indivisibility. The goals here are two fold. First, this section reacquaints the reader with issue indivisibility as an explanation for war. Second, the results verify that the type of indivisibility developed is not inherently peace promoting. Thus, whatever peace-inducing effects that arise later are a function of the competing incentives added.

Two states, A and B, bargain over a basket of goods with a total value standardized to 1. Suppose that $\phi \in [0, 1]$ portion of the good is indivisible; the remaining $1 - \phi$ represents the side payment available. Thus, increasing ϕ increases the value of the indivisible prize and reduces the value of the side payments. For example, ϕ could represent the Spanish throne in 1714, while $1 - \phi$ captures external territories (e.g., Spanish Netherlands, Gibraltar, and Menorca) that Spain could offer as concessions.[1]

I use the ultimatum game to demonstrate the main results, with further analysis in the discussion. The interaction begins with A demanding x. Normally, A could select any portion of the good. Here, the indivisibility limits A's proposal to the disjunction between $[0, 1 - \phi]$ and $[\phi, 1]$.[2]

[1]This was done as a part of the Treaty of Utrecht. See Paine (2021) for a discussion of issues with directly dividing the throne itself.

[2]As such, receiving any part of the good is not a prerequisite for receiving a different part. However, as I explain below, the results are not sensitive to this assumption.

After A proposes x, B accepts or rejects. Accepting implements the division. Payoffs here are x to A and $1 - x$ to B. Rejecting leads to war. As standard, A earns $p - c_A$ and B earns $1 - p - c_B$, where $p \in [0, 1]$ and $c_i > 0$. To avoid corner solutions, I assume that both parties have positive war payoffs throughout.

I use this parameterization of issue indivisibility because of its rigid form. This stacks the deck against indivisibilities helping in the following extensions. For example, I could allow for two or more indivisible goods. However, the presented model's ϕ value treats those indivisible goods as a single item and thus creates a strictly more indivisible environment. This assumption is mostly innocuous, and I describe where it matters.

The standard issue indivisibility problem results from this setup:

Proposition 1. *War occurs in equilibrium if* $\phi > max\{p + c_B, 1 - p + c_A\}$.

Peaceful agreements require both to prefer that deal to war. When ϕ is larger than $p + c_B$, B must receive the indivisible good to prefer peace. Likewise, when ϕ is larger than $1 - p + c_A$, A must receive the indivisible good. But the indivisible good can only go to one player, and thus war occurs. A large indivisible good and a small quantity of side payments (i.e., $1 - \phi$ sufficiently small) therefore imply that bargaining fails. Using succession as the example again, if the value of the monarchy (ϕ) is too large relative to the total value of territorial side payments ($1 - \phi$), both sides prefer war to the other receiving the crown.

3 Indivisibilities and Shifting Power

I now turn to how indivisibilities can promote peace, beginning with a model of preventive war. This is of broad interest because the underlying mechanism for conflict here is a commitment problem, which has applications in comparative politics, civil conflict, and international relations (Powell 2004). Consider a classic two-stage extension of the baseline model. Play begins with A demanding x_1, with the same restrictions that ϕ brought as before. B accepts or rejects. Rejecting initiates a game-ending war. Accepting locks in the settlement for the period and advances to the next stage. There, A demands x_2. As before, B accepts or rejects; this time, either choice ends the game.

Payoffs are as follows. The states share a common discount factor $\delta \in (0, 1)$. If B accepts both demands, A earns $(1-\delta)x_1+\delta x_2$, while B receives $(1-\delta)(1-x_1)+\delta(1-x_2)$.

If B accepts in the first period but rejects in the second, A earns $(1-\delta)x_1 + \delta(p' - c_A)$, while B earns $(1-\delta)(1-x_1) + \delta(1 - p' - c_B)$, where $p' \in (p, 1]$. Finally, if B rejects in the first period, A earns $p - c_A$ while B earns $1 - p - c_B$.

Setting $\phi = 0$, this model generates the standard shifting power commitment problem. In the second period, A's share of the power equals p'. It therefore captures a portion of the good commensurate with that amount. B recognizes that A cannot credibly promise to take less at that stage. Moreover, it realizes that A's share of the power in the first stage is only p. In turn, it may prefer a costly war in the first stage under more favorable terms than an efficient but disadvantageous settlement later.

Nevertheless, indivisibilities introduce a more complicated relationship:

Proposition 2. *In the preventive war model, the relationship between indivisibilities and war is nonmonotonic, with the probability of war minimizing at middling levels of ϕ for some parameters.*

How do indivisibilities promote peace? Consider A's equilibrium demand in the second stage. As the proposer, its goal is to give the least it can to B without sparking a war. Without indivisibilities, it keeps $p' + c_B$ for itself and leaves $1 - p' - c_B$ for B. In turn, war occurs in the first stage if B earns more from fighting than taking everything up front and securing only $1 - p' - c_B$ later. The appendix works through the details. Intuitively, if B is sufficiently patient, securing a larger share of the pie over the long term is worth suffering the inefficiencies of war.

Now consider how indivisibilities alter things. Before, A demanded $p' + c_B$ in the second stage. But if the good is sufficiently indivisible, capturing that exact amount is impossible. Demanding any more results in B initiating A war. The only remaining option is to demand *less*. As long as the good is not too indivisible, A prefers giving itself the smaller share. Because B receives more in the second stage, it must be *more* patient than in the previous case to fight a war in the first stage. Thus, indivisibilities cause peace here because A can credibly commit to greater concessions later. Using the succession example, monarchs have less concern about moderate power shifts—the indivisibility prevents opponents from taking any of the biggest prize, the crown.

Phrased like this, the model reveals a deeper break from conventional wisdom. Powell (2006) notes that wars due to issue indivisibility can be recast as a commitment problem. This remains true here. But that does not imply that every constraint on the

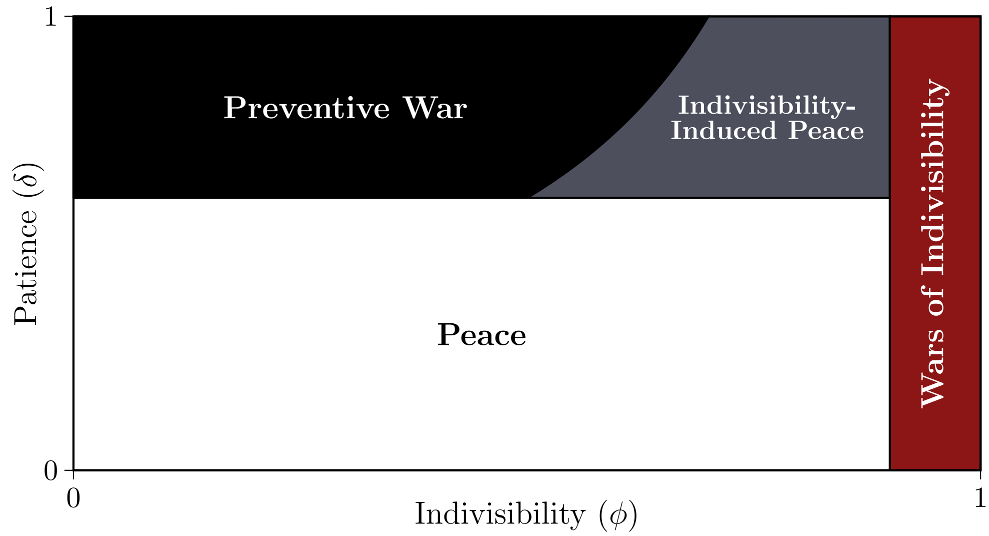

Figure 1: Equilibrium outcome as a function of indivisibilities and patience.

bargaining space is inherently problematic. Here, in fact, those limits render another type of commitment problem harmless. And because both can freely choose war but do not, the issue indivisibility leads to a Pareto welfare improvement.

Figure 1 illustrates the result. The nonmonotonicity arises when patience is high. Without serious indivisibilities, the standard preventive war mechanism takes hold. Increasing indivisibilities forces A to demand less post-shift, resolving the commitment problem. However, extreme indivisibilities make any post-shift agreement untenable, reverting the outcome back to war. Note further that the indivisibilities can be extreme—as the literature argues (Hassner 2003)—and still induce peace. For the sample parameters, the indivisible good can be up to 90% of the overall value and still help. That is, even if outlying territories are only a fraction as valuable as a kingship, indivisibilities may yet be useful. For other distributions of power and costs of war, the threshold could be higher. The binding limitation is that it cannot be fully indivisible.

This result is not sensitive to the modeled indivisibility. The key requirement is simply that the indivisibility prohibits the rising state from taking as much post-shift. This allows a given extensive form to push up against the general "inefficiency condition" that triggers a commitment problem regardless of the bargaining protocol (Powell 2004). Thus, the mechanism arises with multiple indivisible goods or an initial section of the bargaining good that is a prerequisite before offering any side payments. In that light, the result requires A to have some proposal power. If B were to have *all* the proposal power, the extensive form maximizes its negotiation payoff by assumption (Fey and Kenkel 2021), and so indivisibilities could only hurt its post-shift payoff.

4 Indivisibilities and Information Problems

Indivisibilities can also reduce the probability of war when asymmetric information would otherwise cause conflict. In the appendix, I derive the equilibrium of a standard ultimatum crisis bargaining model with uncertainty over the receiver's cost. The only modification is to include the explicit ϕ parameter to measure what portion of the good is indivisible. Once more, the overall relationship is nonmonotonic, with middling indivisibilities sometimes maximizing peace.

The reason is straightforward. Under normal circumstances, the proposer weighs the value of a slightly larger demand against the additional probability of war. The equilibrium proposal strikes the optimal balance. Adding indivisibilities can make that specific demand impossible. Instead, the best feasible demand may be smaller. More types therefore accept, and the probability of war declines. Full indivisibilities still trigger the all-or-nothing problem, which makes the relationship nonmonotonic overall.

Intuitively, indivisibilities can make the optimal proposal obvious and peace-assuring in a way that would not be possible otherwise. The 1954 French cession of its possessions in India, most notably Pondichéry, illustrate the logic. The territory itself was reasonably *de facto* indivisible due to a lack of reasonable ways to split possession of the cities. France was reluctant to let it go (Miles 1995, 57–58). But had France proposed to keep the territory, it would have certainly invited war.[3] The indivisibility therefore made forgoing that part of the bargaining space an obvious choice.

Instead, France aimed to extract concessions that it knew were divisible (Miles 1995, 73–75). Moreover, given the cession to India, there was little doubt that the demands would be acceptable. The "Agreement on De Facto Transfer of French Establishments"

[3]Indeed, when Portugal stood firm over Goa in 1961, India annexed it via invasion. The key difference between France and Portugal was that Portuguese Prime Minister António de Oliveira Salazar was an ardent colonialist, with substantially greater resolve than France. This caused the traditional problem Proposition 1 described, where both sides preferred conflict to losing the indivisible good.

assured free choice of nationality (Article IV), kept constant the local civil service (Article V), guaranteed free travel (Articles V and XV), secured the right to work (Article VII), supplied French business interests (Article XVIII), made France financially whole on outstanding debts (Article XX), and kept the distinctly French character and culture of Pondichéry (Articles XXIV and XXV).

5 Conclusion

This note evaluated the idea that issue indivisibilities inherently promotes conflict. To the contrary, the models presented here show that indivisibilities can induce peace when mixed with other sources of bargaining frictions. As such, it is not surprising that the empirical relationship is inconsistent.

More broadly, this note underscores the importance of taking complete comparative statics and mixing sources of bargaining frictions. Much of our knowledge on conflict comes from simple comparisons of some bargaining friction to static bargaining games with complete information. The real world is more complicated. If we want to better understand how mechanisms for war interact, then we must integrate them into the same model. In matters of war and peace, sometimes two wrongs can make a right.

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6 Appendix

Let $\mathbb{X}$ be defined as the disjunction of the sets $[0, 1-\phi]$ and $[\phi, 1]$—i.e., the feasible set of proposals. The following Lemma will be useful throughout the proof:

Lemma 1. *$x \in \mathbb{X}$ if and only if $\phi \leq max\{x, 1-x\}$.*

The proof is straightforward. Suppose $x \geq \phi$. Then a construction of x is $\phi + (1-\phi)\alpha$—that is, give A the indivisible object and use a portion $\alpha[0, 1]$ of the side payment to reach the desired x. If $\phi > x$, then x is only possible if the side payment covers x, or $1 - \phi \geq x$. Rearranging gives $\phi \leq 1 - x$. This proves the lemma.

6.1 Proof of Proposition 2

In the second stage, B accepts x_2 if:

$$(1-\delta)(1-x_1) + \delta(1-x_2) \geq (1-\delta)(1-x_1) + \delta(1-p'-c_B)$$

$$x_2 \leq p' + c_B$$

There are three cases to consider: (1) A can make that exact proposal, (2) A cannot make that exact proposal but it can propose an alternative that is mutually acceptable, and (3) no mutually acceptable proposal is possible to offer.

6.1.1 The Proposal $x_2 = p' + c_B$ Is Feasible

If $p' + c_B \in \mathbb{X}$, then A demands $p' + c_B$ and B accepts. From Lemma 1, the non-knife edge condition for this is $\phi < max\{p' + c_B, 1 - p' - c_B\}$. I fully solve the game under this condition before exploring the other parameter spaces.

Moving up to the first stage, using $x_2 = p' + c_B$ as the expected division, B accepts x_1 if:

$$(1-\delta)(1-x_1) + \delta(1-p'-c_B) \geq 1 - p - c_B$$

$$x_1 \leq \frac{p - \delta p'}{1-\delta} + c_B \qquad (1)$$

Because x_1 is 0-to-1 constrained, demanding nothing is still insufficient for B to accept if:

$$\frac{p - \delta p'}{1 - \delta} + c_B < 0$$
$$\delta > \frac{p + c_B}{p' + c_B} \tag{2}$$

Thus, if $\phi < max\{p' + c_B, 1 - p' - c_B\}$ and $\delta > \frac{p+c_B}{p'+c_B}$, B prevents in the first period in equilibrium.[1]

By analogous argument, B is willing to accept some value of x_1 if $\delta < \frac{p+c_B}{p'+c_B}$. The calculation of Line 1 showed that the optimal acceptable demand from A's perspective is $\frac{p-\delta p'}{1-\delta} + c_B$.[2] Moreover, A prefers this to war if $(1-\delta)\left(\frac{p-\delta p'}{1-\delta} + c_B\right) + \delta(p' + c_B) > p - c_A$. This reduces to $c_A + c_B > 0$, which is true. Thus, if $\frac{p-\delta p'}{1-\delta} + c_B \in \mathbb{X}$, then A demands that amount. By Lemma 1, the non-knife edge condition for this is $\phi < max\{\frac{p-\delta p'}{1-\delta} + c_B, 1 - \frac{p-\delta p'}{1-\delta} - c_B\}$. As such, if $\phi < max\{p' + c_B, 1 - \frac{p-\delta p'}{1-\delta} - c_B\}$ and $\delta < \frac{p+c_B}{p'+c_B}$, A demands $\frac{p-\delta p'}{1-\delta} + c_B$ in the first period, B accepts, and peace prevails throughout.[3]

If $\phi > max\{\frac{p-\delta p'}{1-\delta} + c_B, 1 - \frac{p-\delta p'}{1-\delta} - c_B\}$, then the largest amount A can demand while still inducing acceptance is $1 - \phi$. This is not acceptable to A if:

$$p - c_A > (1 - \delta)(1 - \phi) + \delta(p' + c_B)$$
$$\phi > \frac{1 - p + c_A - \delta(1 - p' - c_B)}{1 - \delta} \tag{3}$$

As such, if $\delta < \frac{p+c_B}{p'+c_B}$, $\phi < max\{p' + c_B, 1 - p' - c_B\}$, and $\phi > max\{\frac{p-\delta p'}{1-\delta} + c_B, \frac{1-p+c_A-\delta(1-p'-c_B)}{1-\delta}\}$, then war occurs in the first period.[4]

Meanwhile, if Line 3 fails, then A is willing to make the deal. The total conditions on the parameter space for this are $\delta < \frac{p+c_B}{p'+c_B}$, $\phi > max\{\frac{p-\delta p'}{1-\delta} + c_B, 1 - \frac{p-\delta p'}{1-\delta} - c_B\}$, and $\phi < max\{p' + c_B, \frac{1-p+c_A-\delta(1-p'-c_B)}{1-\delta}\}$.[5] Then A demands $x_1 = 1 - \phi$, B accepts, and peace prevails throughout.

[1] There are multiple equilibria here—A is indifferent in its demand choice because B rejects regardless. A similar phenomenon occurs in the other parameter spaces where war occurs in the first period, and so I omit further references to multiple equilibria.

[2] This amount is strictly less than 1 because $\frac{p-\delta p'}{1-\delta} + c_B < 1$ reduces to $\delta(1 - p' - c_B) < 1 - p - c_B$, which is always true.

[3] I can write the condition on ϕ more compactly because $\frac{p-\delta p'}{1-\delta} + c_B < p' + c_B$ and $1 - p' - c_B < 1 - \frac{p-\delta p'}{1-\delta} - c_B$.

[4] I can write the lower bound on ϕ more compactly because $\frac{1-p+c_A-\delta(1-p'-c_B)}{1-\delta} > 1 - \frac{p-\delta p'}{1-\delta} - c_B$.

[5] I can write the upper bound on ϕ more compactly because $\frac{1-p+c_A-\delta(1-p'-c_B)}{1-\delta} > 1 - p' - c_B$.

6.1.2 The Proposal $x_2 = p' + c_B$ Is Not Feasible But Some Mutually Acceptable Proposals Are

In the remaining cases, $\phi > max\{p' + c_B, 1 - p' - c_B\}$. This means that A cannot demand its optimal amount in the second stage. Instead, the largest it can keep for itself while inducing acceptance is $1 - \phi$. A prefers peace if:

$$(1 - \delta)(1 - x_1) + \delta(p' - c_A) < (1 - \delta)(1 - x_1) + \delta(1 - \phi)$$

$$\phi < 1 - p' + c_A \tag{4}$$

From here, the continuation values for the second period are $1 - \phi$ for A and ϕ for B. In turn, B accepts x_1 if:

$$(1 - \delta)(1 - x_1) + \delta\phi \geq 1 - p - c_B$$

$$x_1 \leq \frac{p + c_B - \delta(1 - \phi)}{1 - \delta} \tag{5}$$

Note that B will reject every possible x_1 proposal if:

$$\frac{p + c_B - \delta(1 - \phi)}{1 - \delta} < 0$$

$$\phi < 1 - \frac{p + c_B}{\delta} \iff \delta > \frac{p + c_B}{1 - \phi} \tag{6}$$

This yields the first equilibrium parameter space within this case. If $\phi > max\{p' + c_B, 1 - p' - c_B\}$ and $\phi < min\{1 - \frac{p+c_B}{\delta}, 1 - p' + c_A\}$, war occurs in the first period.

The right hand version of Line 6 is what drives the nonmonotonic relationship. It is a stricter condition to trigger B rejecting all demands, as the condition without relevant indivisibilities was $\delta > \frac{p+c_B}{p'+c_B}$. Because $\phi > 1 - p' - c_B$, B must be more patient to always want to fight a preventive war with indivisibilities.

To demonstrate that peace results here, suppose now that $\phi < 1 - \frac{p+c_B}{\delta}$, meaning that B is willing to be bought off in the first period. From Line 5, the largest demand that A can make while still inducing acceptance is $\frac{p+c_B-\delta(1-\phi)}{1-\delta}$. Suppose that this amount is feasible. From Lemma 1, the condition for this is $\phi < max\{\frac{p+c_B-\delta(1-\phi)}{1-\delta}$ or $\phi < 1 - \frac{p+c_B-\delta(1-\phi)}{1-\delta}\}$. To eliminate cutpoints that are a function of themselves, I rewrite this condition as $p > \delta(1-\phi) + \phi(1-\delta) - c_B$ or $p < 1 - \phi - c_B$.[6]

Meanwhile, A prefers proposing this quantity to war if:

$$(1-\delta)\left(\frac{p + c_B - \delta(1-\phi)}{1-\delta}\right) + \delta(1-\phi) > p - c_A$$

$$c_A + c_B > 0$$

This is true. As such, if $\phi > max\{p' + c_B, 1 - p' - c_B, 1 - \frac{p+c_B}{\delta}\}$, $\phi < 1 - p' + c_A$, and $p > \delta(1-\phi) + \phi(1-\delta) - c_B$ or $p < 1 - \phi - c_B$, A demands $x_1 = \frac{p+c_B-\delta(1-\phi)}{1-\delta}$ in the first period. B accepts, and peace prevails.

Now suppose that $x_1 = \frac{p+c_B-\delta(1-\phi)}{1-\delta}$ is not feasible. A's best alternative is $x_1 = 1-\phi$, and it prefers this to war if:

$$(1-\delta)(1-\phi) + \delta(1-\phi) > p - c_A$$

$$\phi < 1 - p + c_A$$

This is true. The original condition on this parameter space was that A preferred giving ϕ to B in the second period, which mandates $\phi < 1 - p' - c_A$. This is a stricter condition. Intuitively, if A is willing to overlook the indivisibility in the second period when it is stronger, it is certainly willing to overlook the indivisibility in the first period.

Summarizing this last case, if $\phi > max\{p' + c_B, 1 - p' - c_B, 1 - \frac{p+c_B}{\delta}\}$, $\phi < 1 - p' + c_A$, $p < \delta(1-\phi) + \phi(1-\delta) - c_B$, and $p > 1 - \phi - c_B$, A demands $x_1 = 1 - \phi$. B accepts, and peace prevails throughout.

6.1.3 No Mutually Acceptable x_2 Is Feasible

By Line 4, the conditions for this are $\phi > 1 - p' + c_A$ and $\phi > max\{p' + c_B, 1 - p' - c_B\}$. Because $1 - p' + c_A > 1 - p' - c_B$, the condition can be more compactly written as $\phi > max\{p' + c_B, 1 - p' + c_A\}$. War must occur here in the second stage.

Moving up a step, B accepts in the first stage if:

$$(1-\delta)(1-x_1) + \delta(1 - p' - c_B) \geq 1 - p - c_B$$

[6] I put this in terms of p because solving the first inequality for ϕ generates an additional condition based on whether a value is negative or positive (thus determining whether the inequality needs to flip).

$$x_1 \leq \frac{p - \delta p'}{1-\delta} + c_B$$

From Line 2, no proposal appeases B if $\delta > \frac{p+c_B}{p'+c_B}$. Thus, if $\phi > max\{p' + c_B, 1 - p' + c_A\}$ and $\delta > \frac{p+c_B}{p'+c_B}$, B prevents in the first period.

If $\delta < \frac{p+c_B}{p'+c_B}$, then A's most attractive acceptable proposal is $\frac{p-\delta p'}{1-\delta} + c_B$. From Lemma 1, the non-knife edge condition for this quantity to be in $\mathbb{X}$ is $\phi < max\{\frac{p-\delta p'}{1-\delta} + c_B, 1 - \frac{p-\delta p'}{1-\delta} - c_B\}$. Moreover, A prefers this to war if $(1-\delta)\left(\frac{p-\delta p'}{1-\delta} + c_B\right) + \delta(p' - c_A) > p - c_A$. This reduces to $c_A + c_B > 0$, which is true. As such, if $\delta < \frac{p+c_B}{p'+c_B}$, $\phi < max\{\frac{p-\delta p'}{1-\delta} + c_B, 1 - \frac{p-\delta p'}{1-\delta} - c_B\}$, and $\phi > max\{p' + c_B, 1 - p' + c_A\}$, A proposes $x_1 = \frac{p-\delta p'}{1-\delta} + c_B$ and B accepts. Peace prevails in the first stage, but war occurs in the second.

The next case is that A cannot propose $x_1 = \frac{p-\delta p'}{1-\delta} + c_B$, which by Lemma 1 means that $\phi > max\{\frac{p-\delta p'}{1-\delta} + c_B, 1 - \frac{p-\delta p'}{1-\delta} - c_B\}$. The largest alternative that still induces B's acceptance is $x_1 = 1 - \phi$. This is not acceptable to A if:

$$p - c_A > (1-\delta)(1-\phi) + \delta(p' - c_A)$$

$$\phi > 1 - \frac{p - \delta p'}{1-\delta} + c_A$$

Combining terms, if $\phi > max\{1 - \frac{p-\delta p'}{1-\delta} + c_A, p' + c_B\}$ and $\delta < \frac{p+c_B}{p'+c_B}$, then war occurs in the first period.[7]

Finally, if $\phi < 1 - \frac{p-\delta p'}{1-\delta} + c_A$, $\phi > p' + c_B, 1 - p' + c_A, 1 - \frac{p-\delta p'}{1-\delta} - c_B$, and $\delta < \frac{p+c_B}{p'+c_B}$, then A demands $x_1 = 1 - \phi$ and B accepts. Peace prevails in the first stage, but war occurs in the second.

6.2 Incomplete Information Model

Consider the following extension to the baseline model. Nature begins by drawing B's cost of war c_B from the interval $[\underline{c}_B, \overline{c}_B]$ with a twice continuously differentiable cumulative distribution function $F_B(c_B)$ and corresponding probability density function $f_B(c_B)$. I assume that the distribution's hazard rate $\frac{f_B(c_B)}{1-F_B(c_B)}$ is weakly increasing. B sees the draw but A only knows the prior.

Despite the different structure, the main result parallels commitment problems:

[7] I can write the lower bound on ϕ more compactly because $1 - \frac{p-\delta p'}{1-\delta} + c_A > 1 - \frac{p-\delta p'}{1-\delta} - c_B$, $1 - \frac{p-\delta p'}{1-\delta} + c_A > 1 - p' + c_A$, and $p' + c_B > \frac{p-\delta p'}{1-\delta} + c_B$.

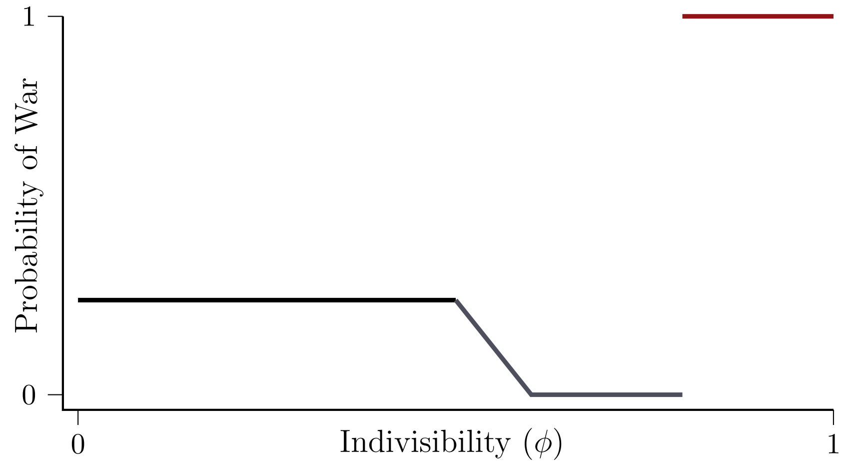

Figure 2: The probability of war as a function indivisibilities.

Proposition 3. *In the asymmetric information model, the relationship between indivisibilities and war is nonmonotonic, with the probability of war minimizing at middling levels of ϕ for some parameters.*

In a perfectly divisible world, A weighs the risk-return tradeoff. Demanding more yields a better agreement whenever B accepts but also increases rejection rates. The proposer therefore must balance the two. Call the optimal balance x^*. If x^* is feasible, A demands it. However, sufficient indivisibilities make proposing x^* impossible. Instead, A must decide between demanding the largest feasible amount less than x^* or the smallest feasible amount greater than x^*. If the lesser demand generates a greater payoff, the probability of war declines.

As Proposition 3 claims and Figure 2 illustrates, A sometimes takes the safer route. In the left half of the figure, indivisibilities do not impact A's ability to demand x^*. As such, the probability of war remains static. Sufficient indivisibilities force A to alter its decision. Choosing the more conciliatory demand, the probability of war declines. In fact, indivisibilities may induce A to make a no-risk proposal. However, in the rightmost portion of the figure, the indivisibilities are extreme. War becomes certain.

6.2.1 Proof of Proposition 3

To begin, a given type of B accepts if $1-x \geq 1-p-c_B$, or $x \leq p+c_B$. As a result, the probability that B accepts any given demand within the interior is $F(x-p)$. Using that, A's objective function is:

$$(p-c_A)F(x-p)+x(1-F(x-p))$$

The first order condition of this is:

$$\frac{\partial}{\partial x}(p-c_A)F(x-p)+x(1-F(x-p))=0$$

$$\frac{1}{x-p+c_A}=\frac{f(x-p)}{1-F(x-p)} \tag{7}$$

If such a solution exists, it is unique. This is because the left hand side strictly decreases in x and the right hand side weakly increases in x. For this reason, the derivative is also increasing from the left and decreasing to the right, and therefore it is a maximizer.[8] Call the unique solution x^*. If x^* is feasible, then A must demand that amount.

If not, then the two feasible demands nearest to and straddling x^* are ϕ and $1-\phi$. Examining whether the probability of war increases or decreases requires checking whether $x=\phi$ or $x=1-\phi$ yields the higher payoff.[9] If $1-\phi$ yields a greater payoff, then the probability of war decreases over the baseline without indivisibilities.[10]

Even if A prefers the smaller demand for moderate indivisibilities, the ultimate relationship is nonmonotonic. If $\phi > max\{p+\underline{c}_B, 1-p+c_A\}$, then A must receive the entire indivisible good to be satisfied. But this leaves an insufficient amount for the lowest cost type to accept. Thus, the probability of war is $F(\phi-p)$, which is positive. Furthermore, if $\phi > max\{p+\bar{c}_B, 1-p+c_A\}$, then no type accepts A's demand of $x=\phi$. The probability of war moves to 1.

[8]If $\frac{1}{c_A+\underline{c}_B} < f(\underline{c}_B)$, then the derivative is negative at $p+\underline{c}_B$, the largest demand with a zero probability of war. This results in a corner solution, where A demands that amount.

[9]One of these must maximize A's optimization problem because the derivative of A's objective function at $1-\phi$ (the amount less than x^*) and all values less than that is positive and the derivative of A's objective function at ϕ (the amount greater than x^*) and all values greater than that is negative.

[10]There is a more rigorous way to examine this question at the exact value of ϕ that makes x^* just feasible. For any given optimization problem, if the first non-zero odd derivative is negative, then A prefers $1-\phi$ to ϕ at that instantaneous point.

6.2.2 A General Result with Uncertainty

Consider the model with uncertainty except now Nature also draws A's cost of war from a common prior distribution with the same requirement as B's.[11] Both draws are privately observed by the state, with the other only knowing the prior. I now show a general result for when the probability of war equals 0 regardless of the proposer under indivisibilities of the type described below but strictly positive without indivisibilities.

First, I show that the probability of war is strictly positive without indivisibilities. This is straightforward. Because a proposer's type does not change a receiver's accept/reject decision, the optimization problem is the same whether there is one-sided or two-sided uncertainty. Drawing from Line 7 suppose that a solution exists to

$$\frac{1}{x_i - p_i + c_i} = \frac{f_{-i}(x_i - p_i)}{1 - F_{-i}(x_i - p_i)}$$

for all $c_i \in [\underline{c}_i, \overline{c}_i]$, where c_i represents the proposer's cost, x_i represents the proposer's share of the settlement, p_i represents the proposer's probablility of winning, and f_{-i} and F_{-i} represent the other state's PDF and CDF. Then, for each state and each realization of that state's cost of war, the equilibrium demand is rejected with positive probability without indivisibilities. By construction, this is true regardless of the identity of the proposer.

This is sufficient for the following result:

Proposition 4. *Suppose indivisibilities are such that* $\mathbb{X}$ *only includes elements from* $[0, p - \overline{c}_A)$, $(p - \underline{c}_A, p + \underline{c}_B)$, *and* $(p + \overline{c}_B, 1]$, *with at least one value within* $(p - \underline{c}_A, p + \underline{c}_B)$. *Then the equilibrium probability of war is 0 regardless of which state is the proposer.*

The proof is straightforward. Regardless of which state is the proposer, any division x within $[0, p - \overline{c}_A)$ or $(p + \overline{c}_B, 1]$ is either worse than war for the proposer or induces the receiver to reject. In contrast, the receiver accepts any demand within the interval $(p - \underline{c}_A, p + \underline{c}_B)$, and the payoff to the proposer is greater than its war payoff. Because such a division exists, one such division must be chosen in equilibrium. If A is the proposer, then it chooses the maximum value in that interval; if B is the proposer, then it chooses the minimum value in that interval. The probability of war is 0 either way.

[11] The CDFs for each state need not be identical.