PS 1514: Exam 2

Name:

Show all your work. No proof, no points.

1 Draws (4 Points)

In the standard complete information setup, all wars ended in complete victory or complete defeat. Thus, there were only two outcomes possible. Now let's add a third. Suppose instead that A wins and takes the entire good with probability p_A, B wins and takes the entire good with probability p_B, and a stalemate occurs with probability $1 - p_A - p_B$. In the event of a stalemate, A receives .6 of the good and B receives .4 of it. Regardless of the outcome, states always pay their war costs as normal.

a) Write each side's expected value for war. (Hint: that there should be *four* components to this: a probability and a payoff for the side winning, a probability and a payoff for the side losing, a probability and a payoff for the stalemate, and the cost of war.)

b) Prove that a mutually preferable peaceful alternative x always exists.

2 Mutual Uncertainty (4 points)

A common misconception is that uncertainty over the likely outcome of conflict will lead to bargaining breakdown. This is not true when both sides face the same uncertainty. Imagine a scenario where the likelihood of victory depends heavily on military cohesion. Yet there is no way to know how cohesive the troops are without actually fighting. Even so, both states believe that B's troops will be cohesive with probability q and uncohesive with probability $1 - q$. In the first case, the A wins with probability p_A; in the second, it wins with probability p'_A. (Essentially, cohesion is good for the B's probability of victory.) Regardless of the cohesion, the actors pay the same costs as before. Note that both sides know exactly what the other knows and nothing more.

a) Write each side's expected value for war. (Hint: It may help to draw a diagram that maps out the probability of each outcome occurring.)

b) Prove that, despite the uncertainty, a mutually preferable peaceful alternative x always exists.

3 Preemptive War with Bargaining (4 Points)

Consider the following preemptive war game with a potential one-sided first strike advantage. State A begins the game by choosing whether to fight or not. If it opts not to, A then makes an offer $x \in [0, 1]$. B observes the offer and accepts or rejects it.

Payoffs are as follows. If A fights immediately, it earns $p' - c_A$ and B earns $1 - p' - c_B$. If A makes an offer that B rejects, A earns $p - c_A$ and B earns $1 - p - c_B$. Finally, if B accepts, A earns x and B earns $1 - x$.

For what values of p does A fight immediately?

4 Per-Period War Costs and Stalling (4 points)

Consider the stalling model with the following modification: the states accrue per-period costs c_A and c_B only during the conflict. To illustrate, this alters A's war payoff to $\frac{(1-\delta_A^T)(q-c_A)+\delta_A^T p}{1-\delta_A}$. Suppose (1) a state has unrealized potential, (2) a state is more patient than its opponent, (3) the state from (1) and (2) is the same, (4) conflict does not resolve immediately, and (5) war is not interminable. Prove that, for sufficiently low costs, no mutually acceptable settlement exists.

5 Secret Armament and War (4 points)

The following question simplifies the mechanism for inefficiency of secret armaments. Consider the following two player, two action, simultaneous move game. State A can either *prevent* or *not prevent*. State B can either *build* or *not build*.

Payoffs are as follows. If A prevents and B builds, A earns $p' - c_A$ and B earns $1 - p' - c_B - k$. If A prevents and B does not build, A earns $p' - c_A$ and B earns $1-p'-c_B$. If A does not prevent and B builds, A earns $p+c_B$ and B earns $1-p-c_B-k$. Finally, if A does not prevent and B does not build, A earns x and B earns $1 - x$.

a) Draw and appropriately label a game matrix.

b) Show that if $p' - p > c_A + c_B + k$, it is not a Nash equilibrium for A to not prevent and B to not build.

c) Use your answer from (b) to make an argument that, if $p' - p > c_A + c_B + k$, the game has an inefficient outcome in expectation no matter what x value the states may negotiate in some (unmodeled) previous stage. (Hint: Invoke Nash's theorem.)