Testing the Robustness of the Bargaining Model of War

1) **Multiple Outcomes.** Before, all wars ended in complete victory or complete defeat. Suppose instead that R wins and takes the entire good with probability p_R, G wins and takes the entire good with probability p_G, and an international intervention occurs with probability $1 - p_R - p_G$. If the intervention occurs, a caretaker government is established that gives .6 of the good to G and .4 of it to R. (In essence, the government achieves a very minor victory.) Regardless of the outcome, states always pay their war costs as normal.

a. Write each side's expected value for war. (One point.)

R's expected utility equals:

$$p_R(1) + p_G(0) + (1 - p_R - p_G)(.4) - c_R$$
$$p_R + .4 - .4p_R - .4p_G - c_R$$
$$.6p_R + .4 - .4p_G - c_R$$

and G's expected utility equals:

$$p_R(0) + p_G(1) + (1 - p_R - p_G)(.6) - c_G$$
$$p_G + .6 - .6p_R - .6p_G - c_G$$
$$.4p_G + .6 - .6p_R - c_G$$

b. Prove that a mutually preferable peaceful alternative x always exists. (One point.)

We need to find an x such that:

$$x \geq .6p_R + .4 - .4p_G - c_R$$

and:

$$1 - x \geq .4p_G + .6 - .6p_R - c_G$$
$$x \leq .6p_R + .4 - .4p_G + c_R$$

Such an x exists if:

$$6p_R + .4 - .4p_G - c_R \leq x \leq .6p_R + .4 - .4p_G + c_R$$
$$6p_R + .4 - .4p_G - c_R \leq .6p_R + .4 - .4p_G + c_R$$
$$c_R + c_G \geq 0$$

This holds because $c_R > 0$ and $c_G > 0$.

2) **Uncertainty.** A common misconception is that uncertainty over the likely

outcome of conflict will lead to bargaining breakdown. This is not true when both sides face the same uncertainty. Imagine a scenario where the likelihood of victory depends heavily on military cohesion. Yet there is no way to know how cohesive the troops are without actually fighting. Even so, both the rebels and government believe that the rebels will be cohesive with probability q and uncohesive with probability $1 - q$. In the first case, the rebels win with probability p_R; in the second, they with with probability p'_R. (Essentially, cohesion is good for the rebels' probability of victory.) Regardless of the cohesion, the actors pay the same costs as before.

a. Write each side's expected value for war. (One point.)

R's expected utility equals:

$$q[p_R(1) + (1 - p_R)(0)] + (1 - q)[p'_R(1) + (1 - p'_R)(0)] - c_R$$
$$qp_R + (1 - q)p'_R - c_R$$

And G's expected utility equals:

$$q[p_R(0) + (1 - p_R)(1)] + (1 - q)[p'_R(0) + (1 - p'_R)(1)] - c_G$$
$$q(1 - p_R) + (1 - q)(1 - p'_R) - c_G$$
$$1 - qp_R - (1 - q)p'_R - c_G$$

b. Prove that, despite the uncertainty, a mutually preferable peaceful alternative x always exists. (One point.)

We need to find an x such that:

$$x \geq qp_R + (1 - q)p'_R - c_R$$

And:

$$1 - x \geq q(1 - p_R) + (1 - q)(1 - p'_R) - c_G$$
$$x \leq qp_R + (1 - q)p'_R + c_G$$

Such an x exists if:

$$qp_R + (1 - q)p'_R - c_R \leq x \leq qp_R + (1 - q)p'_R + c_G$$
$$qp_R + (1 - q)p'_R - c_R \leq qp_R + (1 - q)p'_R + c_G$$
$$c_R + c_G \geq 0$$

This holds because $c_R > 0$ and $c_G > 0$.

c. In class, we saw that *asymmetric* uncertainty about the probability of victory can lead to war. Briefly explain why asymmetric

uncertainty creates bargaining problems but symmetric uncertainty does not. (Two points.)

Despite dividing the outcome into two possibilities, each side's expected utility ultimately remains some probability of winning. (Indeed, setting $p_R = qp_R + (1-q)p'_R$ recovers the original model.) So each actor can only base his war decision over the expected outcome of war, and each player's belief about that expected outcome is identical. In contrast, with asymmetric information, the informed type knows its exact probability of victory. This causes stronger types to require more in negotiations, which forces the uninformed actor to choose between buying off both types or just the weak type. The latter leads to war with positive probability.

3) **Private benefits.** The previous cases all used the unitary actor assumption. This time, suppose the government is a unitary actor but the rebels have a leader who controls his group's decision to go to war. The leader still internalizes the 1 unit of value if the rebel group wins and the cost c_R as before. However, if the rebels win, he derives some private benefit $b > 0$ from being in charge of the government. (This could be because it will boost his ego, give him a longer page on Wikipedia, or ensure a lifetime of steak dinners.) Despite the leader's bias for war, the purpose of this question is to show that peaceful agreements can still work provided that b is not too large.

a. Write the government's and the rebel leader's expected values for war. (One point.)

R's expected utility equals:

$$p_R(1+b) - (1-p_R)(0) - c_R$$
$$p_R + p_R b - c_R$$

And G's expected utility equals:

$$p_R(0) + (1-p_R)(1) - c_G$$
$$1 - p_R - c_G$$

b. What is the maximum value of b such that a peaceful settlement still exists? Hint: Attempt to prove the existence of a peaceful settlement as normal. Then rewrite the final inequality in terms of b. (One point.)

A peaceful settlement exists if an x exists such that:

$$x \geq p_R + p_R b - c_R$$

And:

$$1 - x \geq 1 - p_R - c_G$$
$$x \leq p_R + c_G$$

Combining these:

$$p_R + p_R b - c_R \leq x \leq p_R + c_G$$
$$p_R + p_R b - c_R \leq p_R + c_G$$
$$p_R b \leq c_R + c_G$$
$$b \leq \frac{c_R + c_G}{p_R}$$

so the maximum value of b that permits a peaceful settlement is $\frac{c_R + c_G}{p_R}$.

c. Multiply the inequality in part 3b by p_R and interpret its meaning substantively. That is, explain what it represents in English without using any mathematical symbols. (Two points.)

Manipulating the inequality as instructed gives:

$$p_R b \leq c_R + c_G$$

The left side of the inequality represents the expected personal benefit to the leader in the event of war. The right side of the inequality is the total inefficiency of war. So if the expected benefit is less than the total inefficiency, a settlement still exists.