The Fear of Injury: Explaining Delay in Contract Extensions

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Abstract

Why do we observe delay when sports teams and their players attempt to negotiate contract extensions? We show that a team's uncertainty regarding a player's level of risk aversion causes it. Every time a player takes the field, he risks catastrophic injury which will deny him a large contract in free agency. In a game theoretical model of this bargaining problem, the team begins making offers only the most risk averse players would accept. Over the course of a season, more risk acceptant players credibly reveal their preferences by rejecting offers and risking injury during games. The team then rewards them with larger offers. From a managerial perspective, this model shows that teams should not cut-off negotiations during the season and that risk aversion is an important latent factor to constructing an affordable team.

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1 Introduction

On August 21, 2011, Los Angeles Angels of Anaheim starting pitcher Jered Weaver signed a five year, $85 million contract extension. The deal caught many observers by surprise. Weaver would have been one of the top starting pitchers on the free agent market after the 2012 season. If previous deals involving similar players are any indication, Weaver would have received a far greater per anum offer had he pursued free agency.[1]

Why did Weaver pass over an opportunity to make millions more after the 2012 season? Two days later, Weaver responded to critics by rhetorically asking "how much more money do you need?" (DiGiovanna 2011). Although Weaver could have received millions more as a free agent, those additional dollars could not buy much more enjoyment than the $85 million dollar contract could. Indeed, had Weaver suffered a major injury between then and the 2012 off season, he would not have seen any of that money.

Weaver's choice to accept a small but guaranteed payoff is an example of risk averse decision making. In general, risk averse individuals are willing to accept smaller sums of money than gambling between a great payoff and a horrible payoff, even though the expected value of the gamble is greater than the guaranteed amount. For example, a risk averse individual might prefer accepting a guaranteed contract for $40 million to receiving a $100 million contract half of the time and no money the other half of the time. While the gamble has an expected value of $50 million dollars, a risk averse individual might prefer paying a $10 million premium to ensure he receives at least some money.

Although prior research has investigated how risk averse preferences affect arbitration hearings[2], no study systematically investigates how risk aversion alters the free agent market.[3] As the first step in that direction, we investigate a model of contract negotiations between a risk averse player and a risk neutral team.

There are three main findings. First, as a player grows more risk averse, the contract space increases. A team unwilling to re-sign a player during free agency can nevertheless come to terms with that player earlier in the season. Indeed, no matter how much more money a player would receive on the free agent market from another team than his own team, players always re-sign with the team if they are sufficiently risk averse. Risk aversion, not simply dollar figures, determines which players a team can re-sign.

Second, teams benefit in contract negotiations from some degree of risk of injury. If players face extremely unsafe working conditions, the team does not receive the value of the player very frequently and thus cannot achieve a good

[1] For example, Johan Santana signed a six year, $137 million contract in 2008. A year later, CC Sabathia signed a seven year, $161 million contract. And Cliff Lee signed a five year, $120 million contract in 2011.

[2] See Curry and Pecorino 1993; Faurot and McAliister 1992; Faurot 2001; Marburger and Scoggins 1996; Frederick et al 1998.

[3] Bishop, Finch, and Formby (1990) tackle a related topic, analyzing how players with voting power ought to optimally structure the rules for free agency in the National Football League. We instead focus on bargaining behavior leading up to free agency.

outcome. However, if there is perfect player safety, the team cannot leverage the player's risk aversion against him, as rejecting offers entails no risk. As such, and ignoring morality for the moment, the team's optimal level of player safety for contract extensions falls somewhere in between.

Finally, we may wonder why some players re-sign earlier in the season while others wait until just before free agency to agree to terms. In a traditional bargaining setup (Rubinstein 1982), delay is costly to the actors because it prevents them from enjoying the surplus immediately. Nevertheless, patient types willingly cause delay to credibly signal their resolve and eventually receive a larger share of the surplus (Rubinstein 1985; Fudenberg and Tirole 1983; Fudenberg, Levine, and Tirole 1985).

Delay in sports contracts is not costly in this manner. Before a player hits the free agent market, he and his team negotiate over future seasons. But the player and team cannot actually split any of that surplus until that season begins. Therefore, the usual explanation for delay does not apply to such contracts.

Incomplete information regarding risk preferences provides an alternative mechanism. If the team is uncertain of a player's level of risk aversion, it makes progressively greater contract offers as the season continues. More risk acceptant players credibly prove their tolerance for risk by playing games and potentially suffering an injury. The team can therefore sign the most risk averse players to small contracts while eventually providing larger contracts to the more risk acceptant players.

This model resembles the television game show "Deal or No Deal" in that one actor (the team in this article and the "banker" in the show) tries to screen out more risk averse types by offering smaller amounts while the other actor (the player in this article and the contestant in the show) proves his risk acceptance by deliberately engaging in risky behavior. However, in sports, the team and player commonly wish that the player never actually realizes an injury, as injury destroys the contract space. In contrast, on the show, the actors have adversarial preferences over realized risk; the banker prefers the contestant hit bad outcomes, while the contestant wishes to avoid those outcomes. Additionally, sports contract negotiations differ in that the pace is deliberate and managed by agents, which mitigates the history-dependent decision choices Post et al (2008) find on "Deal or No Deal."

From a business management perspective, this article makes two important contributions. To begin, when selecting players out of drafts, teams usually emphasize athletic ability and talent. While these qualities are necessary to form a winning group, teams ought to seek more risk averse players *ceteris paribus*. Risk averse players re-sign with their team for less money than those who are more risk acceptant. By targeting risk averse players, the team can save more money to pursue free agents from other teams.

Second, teams and player occasionally pause contract renegotiations during the season, ostensibly so that impasses do not negatively affect the player's morale. This model indicates that such a bargaining strategy is inadvisable as a general rule. Indeed, pausing negotiations hurts the team either because it forces it to increase contract offers during the preseason or it requires the team to sign the player during the offseason, at which point it can no longer benefit from the player's risk aversion. Consequently, general managers ought to reconsider this bargaining tactic.

This article proceeds as follows. The next section introduces risk aversion more formally. It then covers the basic contract extension bargaining model in which the player is risk averse and the team is risk neutral before advancing to a version with incomplete information in which the player knows his level of risk aversion but the team does not. The third section discusses important implications of the model, explaining why teams benefit from risk aversion, lack of player safety, and midseason negotiations. A brief conclusion follows.

2 Risk Aversion in Bargaining

In this section, we define a game theoretical bargaining model of contract extensions. We begin by reviewing the logic of risk aversion and its application to sports contracts. Next, we derive the equilibrium of a bargaining game in which the player's level of risk aversion is common knowledge to the player and his team. Delay between the team and its player is non-existent in this model; if the player will re-sign with his team, he does so at the first available opportunity. Lastly, we extend the game to cover situations where the player's level of risk aversion is private information. Given certain parameters, this causes delay in negotiations; the team begins by making smaller offers to screen out more risk averse types before offering larger amounts to less acceptant acceptant players.

2.1 Risk Aversion

To model a player's aversion to risk, suppose his expected utility function is $x^{\frac{1}{\alpha}}$, where x is the dollar value of his contract and $\alpha > 1$. This function is increasing in x concave down for weakly positive values of x, which are the necessary and sufficient conditions for a risk averse expected utility function.[4] Note that as α increases, the player's aversion to risk increases as well. Thus, players with higher values of α are willing to accept smaller contract offers than risk injury.

For example, consider an actor with a level of risk aversion $\alpha = 2$ versus a second actor with a level of risk aversion $\alpha = 5$. Suppose they had two choices: accept x immediately or win \$1,000,000 with probability .9 and \$0 with probability .1. The first person prefers accepting any amount greater than \$810,000 immediately to playing the lottery, is indifferent if the amount equals exactly \$810,000, and prefers the lottery if the amount is less than \$810,000. On the other hand, the second person prefers accepting any amount greater than

[4]The first derivative of $x^{\frac{1}{\alpha}}$ is $\frac{1}{\alpha}x^{\frac{1-\alpha}{\alpha}}$. Since x our analysis restricts x to positive values, the first derivative is positive, and therefore the player's expected utility is increasing in x. The second derivative is $\frac{1-\alpha}{\alpha^2}x^{\frac{1-2\alpha}{\alpha}}$. Note that the coefficient $\frac{1-\alpha}{\alpha^2}$ is negative because $\alpha > 1$, while $x^{\frac{1-2\alpha}{\alpha}}$ remains positive because x is positive in our analysis. Therefore, the second derivative is negative, so the function is concave down.

\$590,490, is indifferent if that amount equals exactly \$590,490, and prefers the lottery if the amount is less than \$590,490.

In contrast, we model the team as risk neutral. We could relax this assumption by supposing the team is merely *less* risk averse than the player and obtain the same results. The assumption that the team is less risk averse than the player is reasonable; after all, the team as an entity naturally evens out its gains and losses over time and with a large pool of players, whereas the player only has a short window of time in his life to make money off sports contracts.

2.2 With Complete Information

Consider a season with four periods: preseason, midseason, offseason, and free agency. Figure 1 illustrates the flow of the game. During the preseason, the team makes an offer $x_p \geq 0$, which the player accepts or rejects. Accepting ends the game. The player receives x_p as his contract size, which translates to an expected utility of $x_p^{\frac{1}{\alpha}}$; the team receives the value of the player if he makes it through the rest of the season without suffering a catastrophic injury. The player does not suffer a catastrophic injury between the preseason and midseason with probability p and likewise does not suffer a catastrophic injury between midseason and the offseason with probability p. Therefore, the team's expected utility of signing the player to a contract worth x_p equals $p^2V - x_p$.

If the player rejects, the game transitions to midseason. In the interim, the player must participate in games and risk injury. As before, with probability p, the player suffers no injury; with probability $1 - p$, the players suffers a catastrophic injury. A catastrophic injury ends the interaction, leaving the player and team with 0 for payoffs. If the player remains uninjured, the team makes a new contract offer x_m, which the player accepts or rejects. Again, accepting ends the interaction. From the current period's risk perspective, the player receives $x_m^{\frac{1}{\alpha}}$ while the team receives $pV - x_m$, the value of the player multiplied by the probability he is not injured between the midseason and postseason minus the cost of the contract.

If the player rejects a second time, the game moves to the offseason. Once more, the player remains uninjured with probability p and suffers an interaction-ending injury with probability $1 - p$. If the player survives, the team makes a final offer. The player accepts or rejects. Accepting ends the game. The player receives x_o while the team receives $V - x_o$.

Rejecting pushes the player into free agency, which we do not model explicitly here. Instead, suppose that the player earns n dollars on the free agent market.[5] The specifics of the free agent subgame prove inconsequential, as n only serves as a baseline amount over which the parties must negotiate. Whether the team re-signs the player during free agency is also trivial. The team can make the

[5] A standard second-price auction–a reasonable approximation for free agency–would obtain this result, where n represents the second greatest value among all teams' values for that player (Fudenberg and Tirole 1991, 10-11). For the complete information game, n could alternatively be the player's expected utility for a non-deterministic free agency subgame.

same offer during the offseason period and re-sign the player at that stage, and the player's value V can incorporate any loss the team experiences if the player signs with a competitor.

From the outset, it might appear that the team will be unable to re-sign a player if the free agent market values the player much more than the team. Proposition 1, however, explains that the team's exclusive negotiation period and the player's risk aversion combine to allow the team to re-sign players it otherwise could not.

> **Proposition 1**: Given a player's free agency contract value n and his value to his team V, there exists a critical level of risk aversion α^* for which the player always re-signs with his team if his level of risk aversion is at least α^*.

Put differently, a highly risk averse player prefers taking a smaller but guaranteed contract today to risking injury and receiving a larger contract offer in the future. Since teams are risk neutral, they are more willing to take a risk by signing the player to a guaranteed contract early on, even though he may become injured over the course of the season, in turn rendering the contract a bust. This difference in risk acceptance opens up a larger contract space, allowing the team to re-sign players that it could not if those players were risk neutral. Counter-intuitively, no matter how much more money the player will earn on the free agent market from other teams, he will still re-sign with his team if he is sufficiently risk averse.

Proving this claim first requires the solution to the game. Since this is an extensive form game of complete information, the appropriate solution concept is subgame perfect equilibrium. A subgame perfect equilibrium is a set of strategies such that those strategies are Nash equilibria in every subgame. Specifically, subgame perfect equilibrium requires actors to only make (and believe) credible threats. Throughout this model and the incomplete information version, we assume the player accepts when indifferent between accepting and rejecting.

Let n be the value of the winning offer in the second price auction. In the offseason period, the player accepts any offer $x_o \geq n$ and rejects $x_o < n$. Since the team's payoff is strictly decreasing in x_o, its optimal acceptable offer is $x_o = n$. Thus, the team prefers to make the optimal acceptable offer if:

$$v - n \geq 0$$

$$v \geq n$$

In words, the team makes an offer if its value for the player is at least as great as the value of the player on the free agent market. Risk aversion does not play a role here, as the player does not risk injury from the offseason to the free agency period.

Now consider the midseason period. If the player rejects the offer, he receives n with probability p and 0 with probability $1 - p$. Thus, from the perspective of the midseason period, the player's expected utility for rejecting is:

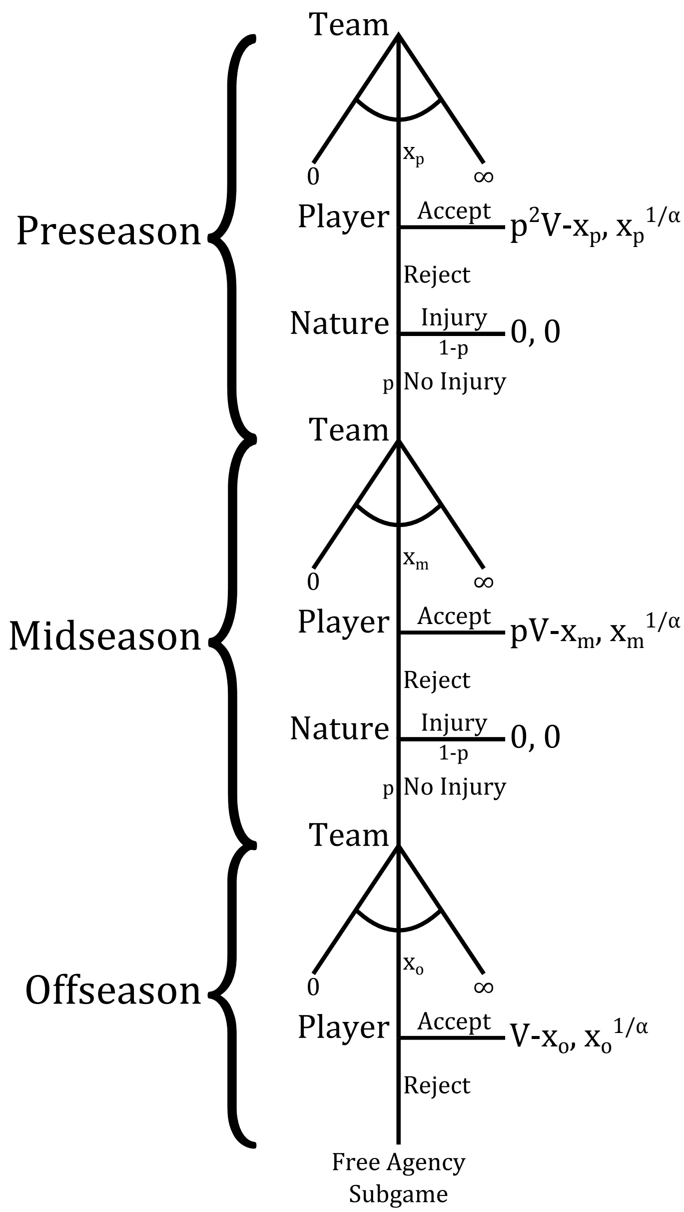

Figure 1: The extensive form of the complete information game.

$$p(n)^{\frac{1}{\alpha}} + (1-p)(0)^{\frac{1}{\alpha}} = pn^{\frac{1}{\alpha}}$$

Thus, the player accepts the contract offer if:

$$x_m^{\frac{1}{\alpha}} \geq pn^{\frac{1}{\alpha}}$$
$$x_m \geq p^{\alpha} n$$

The team's payoff is strictly decreasing in x_m. Therefore, its optimal acceptable offer is $x_m = p^{\alpha} n$.

There are two cases to consider. First, suppose the team optimally makes an acceptable offer in the offseason period. Then the team makes the optimal acceptable midseason offer if:

$$pV - p^{\alpha} n \geq p(V - n) + (1-p)(0)$$
$$1 \geq p^{\alpha - 1}$$

Since $p \in (0, 1)$ and $\alpha > 1$, $p^{\alpha-1}$ is strictly less than 1, so the inequality holds. Thus, if the team is willing to re-sign the player during the offseason, it is willing to re-sign the player midseason.

Second, suppose the team optimally does not re-sign the player during the offseason. Then it makes the optimal acceptable midseason offer if:

$$pV - p^{\alpha} n \geq 0$$
$$V \geq p^{\alpha-1} n$$

Note that $p^{\alpha-1}n$ is less than n. Thus, the team is willing to re-sign players with valuations $V \in [p^{\alpha-1}n, n)$ during midseason that it is not willing to re-sign during the offseason. Essentially, the team's risk neutral preferences and the player's risk averse preferences interact to expand the contract space. In the offseason, there is no risk in contract offers, so the player's risk aversion is irrelevant. But during midseason, the player is willing to accept less money, knowing that catastrophic injury will leave him with no offer whatsoever. In turn, the team is willing to sign the player at the reduced rate even though it would not re-sign the player during the offseason. While the contract will be a bust if the player is injured in the interim, the cheaper, risk-reduced cost makes the team willing to accept the gamble.

Finally, consider the preseason period. There are two cases to consider. First, suppose the team and player will reach an agreement at midseason. Then if the player rejects in the preseason, he receives a contract valued $p^{\alpha} n$ with probability p and 0 with probability $1 - p$. Thus, he accepts x_p if:

$$x_p^{\frac{1}{\alpha}} \geq (p)(p^{\alpha} n)^{\frac{1}{\alpha}} + (1-p)(0)^{\frac{1}{\alpha}}$$
$$x_p \geq p^{2\alpha} n$$

Since the team's expected utility is strictly decreasing in x_p, its optimal acceptable offer is $x_p = p^{2\alpha} n$. If the team offers $x_p < p^{2\alpha} n$, the player rejects, and the team earns $pV - p^{\alpha}n$ with probability p and 0 with probability $1-p$. If the player accepts, the team receives the value of the player with probability p^2 but pays the cost of the contract regardless. Therefore, the team makes the acceptable offer if:

$$p^2V - p^{2\alpha}n \geq p(pV - p^{\alpha}n) + (1-p)(0)$$
$$p^{\alpha} > p^{2\alpha}$$

Since $p \in (0,1)$ and $\alpha > 1$, this inequality holds. So the team makes the optimal offer.

Now suppose the team is unwilling to sign the player at midseason.[6] As such, if the player rejects x_p, he earns n with probability p^2 and 0 with probability $1-p^2$. Thus, he accepts x_p if:

$$x_p^{\frac{1}{\alpha}} \geq p^2(n)^{\frac{1}{\alpha}} + (1-p)^2(0)^{\frac{1}{\alpha}}$$
$$x_p \geq p^{2\alpha}n$$

Since the team's expected utility is strictly decreasing in x_p, its optimal acceptable offer is $x_p = p^{2\alpha}n$. If the team offers $x_p < p^{2\alpha}n$, the player rejects, and the team earns 0. If the player and team come to terms, the team earns V with probability p^2 but pays the cost of the contract regardless. Therefore, the team is willing to make the optimal acceptable offer if:

$$p^2V - p^{2\alpha}n \geq 0$$
$$V \geq p^{2(\alpha-1)}n$$

So if $V \geq p^{2(\alpha-1)}n$, the team makes the optimal acceptable offer. Otherwise, the team makes an unacceptable offer and induces the player to reject. Note again that the team is willing to sign some types of players during the preseason it is unwilling to sign at midseason or during the offseason. To see this, recall that the team only re-signs players with values greater than or equal to $p^{\alpha-1}n$ after the preseason. But during the preseason, it is willing to re-sign players with values greater than or equal to $p^{2(\alpha-1)}n$. This is a lower threshold than $p^{\alpha-1}n$, as $p^{2(\alpha-1)}n < p^{\alpha-1}n$ reduces to $p < 1$, which is true by definition.

Therefore, in equilibrium, the team re-signs a player if $V \geq p^{2(\alpha-1)}n$. Solving for α yields the critical level of risk aversion α^* Proposition 1 establishes:

$$V = p^{2(\alpha^*-1)}n$$
$$\alpha^* = 1 + \frac{ln(v) - ln(n)}{2ln(p)}$$

Thus, if the player's level of risk aversion α is at least α^*, he re-signs with his team; although he would receive a (perhaps substantially) greater offer from another team during free agency, the risk of injury in the interim dictates that the player prefers to re-sign for a lesser amount.

[6] This implies that the team is also unwilling to sign the player during the offseason, which is why there are only two cases to consider.

2.3 With Information Asymmetries

The complete information model generates a further puzzle. The subgame perfect equilibrium has a no-delay property; that is, if the team re-signs the player, it does so during the preseason. However, players and teams often settle contract negotiations during the season, while other players wait to re-sign until the offseason. The complete information account provides no explanation for this behavior.

Relaxing the complete information assumption resolves the discrepancy between the model and negotiations in practice. In this section, we develop a bargaining game of incomplete information. While a player knows his level of risk aversion, teams cannot easily observe this intrinsic preference. Moreover, players have strategic incentives to underrepresent their level of risk aversion; after all, more risk acceptant players receive greater contract offers.

As such, a natural extension to the model incorporates the team's uncertainty about the player's level of risk aversion. This section formally explores such a dynamic. For certain parameters, delay occurs in equilibrium; the team makes stingy offers that only the most risk averse types of players would accept before making more generous offers that less risk averse types accept.

To introduce uncertainty into the interaction, suppose nature begins the game by drawing the player's type from a common prior distribution. The player has a level of risk aversion α_L with probability q_L, a level of risk aversion α_M with probability q_M, and a level of risk aversion α_H with probability q_H, where $\alpha_H > \alpha_M > \alpha_L > 1$ and $q_1 + q_M + q_H = 1$. Greater values of α indicate greater levels of risk aversion. After nature selects the player's type, the player sees his type but the team does not.

Since this is an extensive form game with incomplete information, the appropriate solution concept is perfect Bayesian equilibrium.[7] A perfect Bayesian equilibrium is a set of sequentially rational strategies based off of sequentially rational beliefs. Specifically, this means that the team must update its beliefs about the player through Bayes' rule wherever possible.

Depending on the values of $\alpha_L, \alpha_M, \alpha_H, q_L, q_M, q_L, p, n$, and V, this interaction can have many different equilibrium outcomes. Since we are interested in delay in bargaining, we narrow our focus to a set of parameters in which the following occurs in equilibrium: only the highly risk averse type accepts the team's offer in the preseason, only the medium risk averse type accepts during midseason, and only the least risk averse type accepts during the offseason. If such parameters exist, then we have a causal mechanism for delay in bargaining.

Proposition 2 summarizes the result:

> **Proposition 2**: If $V > n$ and q_L falls in a certain range relative to the other parameters, then delay occurs in a perfect Bayesian equilibrium that survives the D1 refinement.

In particular, the range is $q_L \in (max\{\frac{q_M(p^{\alpha_L+\alpha_M} - p^{1+\alpha_M}) + q_H(p^{\alpha_L+\alpha_M} - p^{\alpha_M+\alpha_H})}{p^2 - p^{1+\alpha_L}},$

[7] For a full treatment of perfect Bayesian equilibrium, see Fudenberg and Tirole 1991.

$\frac{q_M(p^{2\alpha_L}-p^{1+\alpha_M})+q_H(p^{2\alpha_L}-p^{\alpha_M+\alpha_H})}{p^2-p^{2\alpha_L}}\}, q_M\frac{p^{\alpha_L}-p^{\alpha_M}}{p-p^{\alpha_L}})$. The upper bound ensures the team would want to separate out the middle versus the low risk averse types at midseason, while the lower bound ensures that the team would want to separate out the high type from the other two types during the preseason.

If q_L falls in this critical range, the team offers $x_p^* = p^{\alpha_H+\alpha_M}n$ during the preseason, $x_m^* = p^{\alpha_M}n$ at midseason, and $x_o^* = n$ during the offseason. Only the most risk averse type accepts during the preseason, only the medium risk averse type accepts at midseason, and only the least risk averse type accepts during the offseason. Therefore, at midseason, the team believes it is facing a high type with zero probability, the medium type with probability $\frac{q_M}{q_M+q_L}$, and the low type with probability $\frac{q_L}{q_M+q_L}$. Additionally, during the offseason, the team believes it is facing the high or medium type with probability 0 and the low type with probability 1. The team effectively screens out the more risk averse players by forcing them to play games and risk injury, which in turn proves their greater acceptance of risk.

Showing the following is sufficient to prove that these strategies form a perfect Bayesian equilibrium:

1. It is optimal for the least risk averse type to accept x_o^* during the offseason.
2. It is optimal for the medium risk averse type to accept x_m^* at midseason but not optimal for the least risk averse type to accept.
3. It is optimal for the most risk averse type to accept x_p^* during the preseason but it is not optimal for the less risk averse types to accept.
4. It is optimal for the the team to make an offer during the offseason that the least risk averse type is willing to accept.
5. It is optimal for the team to make an offer at midseason that the medium risk averse type is willing to accept but the least risk averse type is not willing to accept.
6. It is optimal for the team to make an offer during the preseason that only the most risk averse type would accept.

We will prove each of these in order. First, we must show that the least risk averse type is willing to accept $x_o = n$ during the offseason. Despite the introduction of asymmetric information, the modified game has the same optimal strategies as the complete information version from the offseason stage forward. If bargaining ever reaches the offseason, uncertainty over the player's risk aversion becomes irrelevant. At that point, the player no longer faces any lotteries over outcomes; explicitly, all three types of players are willing to accept the same offers for x_o that the other types are. Thus, the player accepts the team's offer x_o if $x_o \geq n$ and rejects otherwise. In equilibrium, $x_o = n$. Therefore, the least risk averse type is willing to accept during the offseason.

Second, we must show that it is optimal for the medium risk averse type to accept at midseason but not optimal for the least risk averse type to accept. In equilibrium, the team offers $x_m^* = p^{\alpha_M} n$. If a player rejects the team's contract offer x_m, it earns $n^{\frac{1}{\alpha_i}}$ with probability p and 0 with probability $1-p$, where α_i indicates the respective type's level of risk aversion. Therefore, the player accepts if and only if:

$$x_m^{\frac{1}{\alpha_i}} \geq (p)(n^{\frac{1}{\alpha_i}}) + (1-p)(0)^{\alpha_i}$$
$$x_m \geq p^{\alpha_i} n$$

When the team offers $x_m^* = p^{\alpha_M} n$ in equilibrium, the middle risk averse type is willing to accept because $p^{\alpha_M} n = p^{\alpha_M} n$. In contrast, the least risk averse type rejects, as $p^{\alpha_M} n \geq p^{\alpha_L} n$ reduces to $\alpha_L > \alpha_M$, which is a contradiction. Therefore, the middle risk averse type is willing to accept but the least risk averse type is not.

Third, we must show that it is optimal for the most risk averse type to accept during the preseason but it is not optimal for the less risk averse types to accept. In equilibrium, the team offers $x_p^* = p^{\alpha_H + \alpha_M} n$. If the high type rejects, it could accept x_m^* or reject again and accept x_o^*. In the former case, he earns $p(x_m^*)^{\frac{1}{\alpha_H}}$; in the latter case, he earns $p^2(x_o^*)^{\frac{1}{\alpha_H}}$. Accepting x_m^* is superior if:

$$p(p^{\alpha_M} n)^{\frac{1}{\alpha_H}} > p^2(n)^{\frac{1}{\alpha_H}}$$
$$p^{\alpha_H + \alpha_M} > p^{2\alpha_H}$$
$$p^{\alpha_M} > p^{\alpha_H}$$

This holds. Therefore, if the most risk averse type rejects x_p^*, the best he can do is accept x_m^* at midseason.

Now consider the most risk averse type's decision whether to accept x_p^*. If he accepts, he earns $(x_p^*)^{\frac{1}{\alpha_H}}$. If he rejects, he earns $p(x_m^*)^{\frac{1}{\alpha_H}}$ in expectation. As such, he is willing to accept if:

$$(p^{\alpha_H + \alpha_M} n)^{\frac{1}{\alpha_H}} \geq p(p^{\alpha_M} n)^{\frac{1}{\alpha_H}}$$
$$p^{\alpha_H + \alpha_M} n \geq p^{\alpha_H + \alpha_M} n$$

Thus, the most risk averse type is willing to accept.

However, the less risk averse types are not. First, consider the medium risk averse type's decision. If he accepts, he earns $(x_p^*)^{\frac{1}{\alpha_M}}$. If he rejects, he accepts x_m^* at midseason and earns $p(x_m^*)^{\frac{1}{\alpha_M}}$ in expectation. Consequently, he must reject if:

$$p(p^{\alpha_M} n)^{\frac{1}{\alpha_M}} > (p^{\alpha_H + \alpha_M} n)^{\frac{1}{\alpha_M}}$$
$$p^{2\alpha_M} > p^{\alpha_H + \alpha_M}$$
$$p^{\alpha_M} > p^{\alpha_H}$$

This holds once again. So the middle risk averse type must reject. The same steps show that the least risk averse type must reject the first offer as well. Thus, only the most risk averse type can accept the team's offer in the preseason.

Fourth, we must show that it is optimal for the the team to make an offer during the offseason that the least risk averse type is willing to accept. Since the three types of the player are action equivalent in the offseason stage, the team has complete knowledge of how the player will respond to all possible offers and can therefore appropriately tailor its offer. The team's payoff is strictly decreasing in x_o, so its optimal acceptable offer is $x_o = n$. Alternatively, the team could demand a smaller amount and induce the player to reject, earning the team 0. Thus, the team prefers making the acceptable offer if

$$v - n \geq 0$$
$$v \geq n$$

This result is exactly the same as in the complete information case. However, for the purposes of this proof, the parameters of Proposition 2 guarantee that $v > n$. Consequently, if the game ever reaches the offseason, the team optimally offers $x_o^* = n$, which the least risk averse type is willing to accept in turn.

Fifth, we must show that it is optimal for the team to make an offer at midseason that the medium risk averse type is willing to accept but the least risk averse type is not willing to accept. In equilibrium, the most risk averse type accepts the team's offer during the preseason, so the team's posterior belief that it is facing a most risk averse type of individual is 0. The two least risk averse types reject the offer in the preseason. Therefore, the team's posterior belief it is facing a middle risk averse type is $\frac{q_M}{q_M+q_L}$, while its posterior belief it is facing a least risk averse type is $1 - \frac{q_M}{q_M+q_L} = \frac{q_L}{q_M+q_L}$.

Only three types of offers could be optimal for the team given these constraints: $x_m < p^{\alpha_M} n$, $x_m = p^{\alpha_M} n$, or $x_m = p^{\alpha_L} n$. Any $x_m < p^{\alpha_M} n$ induces both types to reject; $x_m = p^{\alpha_M} n$ induces only the middle type to accept, and $x_m = p^{\alpha_L} n$ induces both types to accept.[8]

We can eliminate $x_m < p^{\alpha_M} n$ from the discussion by showing that $x_m = p^{\alpha_L} n$ is always better for the team. Offering $x_m < p^{\alpha_M} n$ leads to assured agreement if the game reaches the offseason, at which point the player signs for $x_o^* = n$. From the perspective of midseason, the team's payoff for this outcome is $p(V - n)$.

Alternatively, the team could offer $x_m = p^{\alpha_M} n$. With probability $\frac{q_M}{q_M+q_L}$, the player is the middle risk averse type and re-signs, giving the team a payoff of $pV - p^{\alpha_L} n$. With probability $\frac{q_L}{q_M+q_L}$, the player is the least risk averse type and rejects. With probability p, that player does not get injured and re-signs for n. Thus, the team's expected utility for offering $x_m = p^{\alpha_M} n$ is:

[8] Any $x_m \in (p^{\alpha_M} n, p^{\alpha_L} n)$ cannot be optimal; only the middle risk averse type accepts this offer, but the team can profitably deviate by offering a slightly smaller amount and still induce the middle risk averse type (but not the least risk averse type) to accept. Likewise, any $x_m > p^{\alpha_L} n$ cannot be optimal either; both types accept this offer, but the team can profitably deviate to a slightly smaller amount and still induce both types to accept.

$$\frac{q_M}{q_M+q_L}(pV-p^{\alpha_L}n)+\frac{q_L}{q_M+q_L}[p(V-n)]$$

As such, the team strictly prefers offering $x_m = p^{\alpha_M}n$ to $x_m < p^{\alpha_M}n$ if:

$$\frac{q_M}{q_M+q_L}(pV-p^{\alpha_M}n)+\frac{q_L}{q_M+q_L}[p(V-n)] > p(V-n)$$
$$(pV-p^{\alpha_M}n) > p(V-n)$$
$$p > p^{\alpha_M}$$

This holds. Therefore, regardless of the specific values of q_M and q_L, the team always prefers offering the contract size that only the middle risk averse type will accept to offering a contract size that neither type would accept.

Now we can check whether the team prefers offering $x_m = p^{\alpha_M}n$ or $x_m = p^{\alpha_L}n$. As before, if the team offers $x_m = p^{\alpha_M}n$, it earns $\frac{q_M}{q_M+q_L}(pV - p^{\alpha_L}n) + \frac{q_L}{q_M+q_L}[p(V-n)]$. On the other hand, if the team offers $x_m = p^{\alpha_L}n$, both types accept, and the team earns $pV - p^{\alpha_L}n$. Consequently, the team strictly prefers offering $x_m = p^{\alpha_L}n$ if:

$$\frac{q_M}{q_M+q_L}(pV-p^{\alpha_M}n)+\frac{q_L}{q_M+q_L}[p(V-n)] > pV-p^{\alpha_L}n$$
$$p^{\alpha_L} > \frac{q_M}{q_M+q_L}p^{\alpha_M}+\frac{q_L}{q_M+q_L}p$$
$$q_L(p^{\alpha_L - p}) > q_M(p^{\alpha_M}-p^{\alpha_L})$$
$$q_L < q_M\frac{p^{\alpha_L}-p^{\alpha_M}}{p-p^{\alpha_L}}$$

Proposition 2 requires that q_L be strictly less than this figure. Therefore, the team optimally offers an amount only the middle risk averse type accepts.

Sixth, and lastly, we must show that it is optimal for the team to make an offer during the preseason that only the most risk averse type would accept. This is the most complicated of the six steps, as it requires showing how the team would act during midseason and the offseason if it tinkered with its offer in the preseason. Consequently, the appendix details the full proof. Intuitively, if the difference in risk aversion of the three types is big enough and the high and middle types make up enough of the population, the team has incentive to space out its offers. The appendix merely derives the lower bounds for q_L to ensure that this is the team's optimal strategy.

Note that the equilibrium presented here survives the D1 refinement, which requires the team to believe it is facing the type of player with who gains most frequently given the opponent's best response to possible off the equilibrium path strategies of the original player.[9] Here, that requires the team to believe it is facing the least risk averse type off the equilibrium path, since the least risk

[9]See Banks and Sobel 1987 and Cho and Kreps 1987.

averse type is willing to accept the smallest set of all possible offers.[10] Without the D1 refinement, there exist trivial equilibria in which the team offers the risk-reduced equivalent contract for the high type in all periods. Given the team's strategy, it is optimal for all types of the player to accept the team's first offer. Meanwhile, the team's offers in future periods are rational given that those periods are off the equilibrium path and the team can therefore assign the updated belief that the player is a high type with sufficiently high probability, which in turn rationalizes the initial offer. However, these equilibria do not exhibit any sort of learning behavior and are thusly unfulfilling.

3 Discussion

Having described equilibrium outcomes in two bargaining games with risk averse players, we turn to interpreting the results. In particular, we focus on three topics. First, we show that teams can benefit from *decreasing* player safety, at least during contract extension negotiations. Second, we demonstrate that selecting risk averse players during drafts can save teams substantial amounts of money as that player approaches free agency. Finally, we argue that cutting off in-season contract negotiations is a poor bargaining tactic, as it prevents the team from using the screening mechanism detailed in the incomplete information model.

3.1 Perverse Incentives in Injury Management

On first thought, it appears that increasing player safety is an unequivocally positive goal for all parties involved; players do not have to worry about physical harm, while teams ensure their investments do not instantly go bust. However, during the contract renegotiation process, the team preys on the player's risk aversion to drive down contract costs. As player safety decreases, the team can sign players more often and for less money. Consequently, teams *benefit* from the risk of injury to some degree.

To see this, recall that the team re-signs the player if $V \geq p^{2(\alpha-1)}n$ in the complete information model. Taking the first derivative of $p^{2(\alpha-1)}n$ with respect to p yields $2(\alpha-1)p^{2\alpha-3}n$, which is always positive for $\alpha > 1$. Therefore, the threshold is increasing in p, the measure of player safety. Higher rates of injury consequently leave the team in better position to re-sign its players.

Moreover, if the team re-signs the player in equilibrium, its expected utility has a non-monotonic relationship with p. Specifically, for sufficiently low values of p, the team's expected utility increases; but after a critical point, the team's expected utility *decreases*. In other words, the team performs the best in contract negotiations when players do not enjoy complete safety.

[10] In the process of proving the sixth step for Proposition 2, the appendix formally proves that a more risk averse type accepts any offer that a less risk averse type accepts, but a less risk averse type accepts offers a more risk averse type does not.

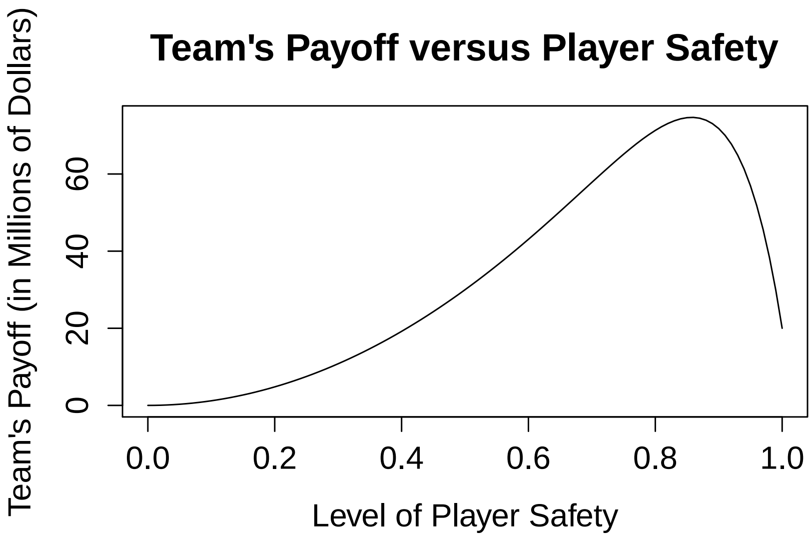

Figure 2: The team's payoff as a function of p for a given set of parameters. If player safety is low, the team achieves a greater payoff as safety increases. However, once safety is sufficiently high, the team fares worse in contract negotiations as safety further increases.

For proof, recall that the team earns $p^2V - p^{2\alpha}n$ if it re-signs the player. Taking the first derivative of $p^2V - p^{2\alpha}n$ with respect to p yields $2pV - 2\alpha p^{2\alpha-1}n$. In turn, the team's expected utility is decreasing in p if:

$$2pV - 2\alpha p^{2\alpha-1}n < 0$$
$$\alpha > \frac{V}{p^{2(\alpha-1)}n}$$

The team's ability to re-sign the player in this instance implies that $V \geq p^{2(\alpha-1)}n$. Thus, for sufficiently large values of α, the team would earn a larger payoff if the player were slightly more likely to get injured.

Figure 2 illustrates this non-monotonic relationship, with $V = \$120,000,000$, $n = \$100,000,000$, and $\alpha = 6.5$. The x-axis varies p, the probability the player does not get injured after each period; higher values of p represent greater player safety. The y-axis is the team's expected utility for the game in millions of dollars, which is $p^2V - p^{2\alpha}n$. When player safety is extremely poor, the team rarely ever enjoys the benefit V and thus benefits from reducing the probability of injury. However, once the probability of surviving a period without injury reaches a critical point, the team earns less as player safety increases because it can no longer adequately take advantage of the player's risk aversion. In the example, the critical point is $p = \frac{V}{\alpha n}^{\frac{1}{2(\alpha-1)}}$, or approximately 0.858. Put

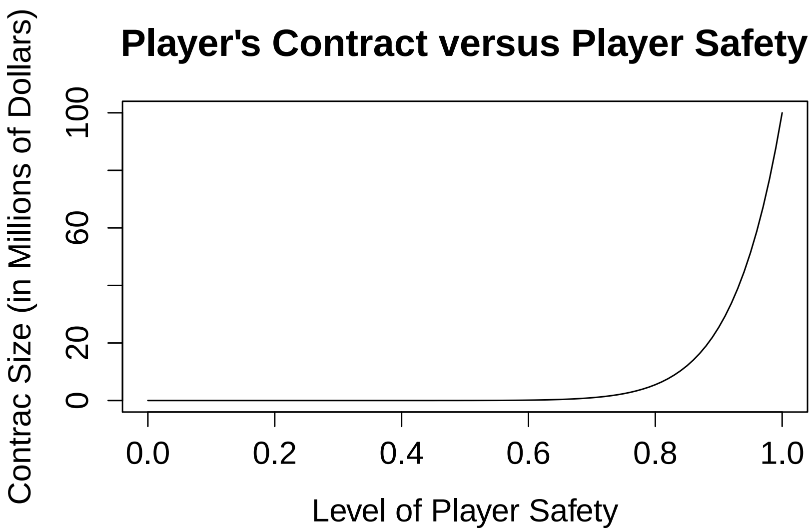

Figure 3: The player's payoff as a function of p for the same set of parameters. If player safety is high, the team cannot leverage his risk aversion against him, and he receives a large contract.

differently, if the player expects to be injured with probability less than about 0.264 over the course of the entire season, team owners surprisingly benefit from making injuries more frequent. Moreover, the team fares worse when players are perfectly safe than when players suffer an injury half of the time during just one half of the season.

This result does *not* imply that teams should create more hazardous conditions for their players. Indeed, increasing the risk of injury creates a large number of externalities not present in the model. For example, if a player already under a long-term contract becomes injured, the team must still pay the contract. Likewise, any sports organization with a moral compass would not want to make such changes out of a sense of altruism. Nevertheless, teams extract additional value out of contracts because some injuries are unavoidable no matter the precautions.

On the other hand, the player's payoff is strictly increasing in p. Figure 3 illustrates the relationship, maintaining the same parameters as Figure 2. While the player receives a large contract size for high levels of player safety, it quickly descends to nothing as the level of player safety decreases. Players only benefit from increasing work safety.

3.2 The Additional Value of Risk Averse Players

When selecting players in a draft, teams normally use performance characteristics such as speed, strength, throwing ability, and potential upside to make their selections. In contrast, this model highlights how risk aversion is an important trait over the long-term that teams should value. Indeed, drafting players with greater levels of risk aversion can potentially save the team millions of dollars as that player approaches free agency.

For example, suppose $n = 100,000,000$, $p = .99$, and $V \geq 100,000,000$. In this case, if the player ever reaches the free agent market, he will sign a contract worth \$100 million, but will get injured over the course of the season with probability $(.99)(.99) = .9801$. Since v is greater than or equal to \$100 million, the team will want to re-sign the player. The only question what price it will have to pay.

Suppose, for simplicity, that two players with identical physical characteristics exist. However, one player's level of risk aversion is $\alpha = 1$ while the other's is $\alpha = 6.5$. In the first case, the player is risk neutral. Therefore, to re-sign the player in the preseason, the team must offer the player his expected value for playing out the season, or $p^{2(1)}n = \$98,010,000$. In the second case, when the player is risk averse at the level $\alpha = 6.5$, the team can re-sign the player in the preseason for $p^{2(6.5)}n$, which less than \$88 million. As a result, if the team drafts the more risk averse player, it can save \$10 million in the long run. The team could then use that extra salary to sign higher quality players on the free agent market than it could had it drafted the risk neutral player instead.

Figure 4 plots the equilibrium preseason contract offer size using those parameters and varying the level of risk aversion α. As in the example, when $\alpha = 1$, the team must offer \$98,010,000 to re-sign the player. But as α increase, the size of the contract decreases. By the team α reaches 50, the team can re-sign the player for under \$40 million.

3.3 The Benefits of Mid-Season Negotiations

In many instances, players and teams cease negotiations during the season. That is, if the team and player fail to reach an agreement during the preseason, the parties completely suspend talks until after the season ends. For example, the St. Louis Cardinals failed to continue negotiations with Albert Pujols in 2011 and Yadier Molina in 2012 (Strauss 2012). Likewise, the New York Mets did not communicate with Jose Reyes in 2011 or David Wright in 2012 (Puma 2012). Taking things to the extreme, the New York Yankees generally do not negotiate with players in-season (Matthews 2012).

This model questions the wisdom of suspending negotiations. The team benefits heavily from fully screening out each type of player. After all, each successive contract offer is larger. More risk averse types accept earlier amounts because they will not receive more valuable offers in the future after factoring in the level of risk. Meanwhile, the team benefits by not mistakenly signing a highly risk averse player to a larger contract that he would be willing to.

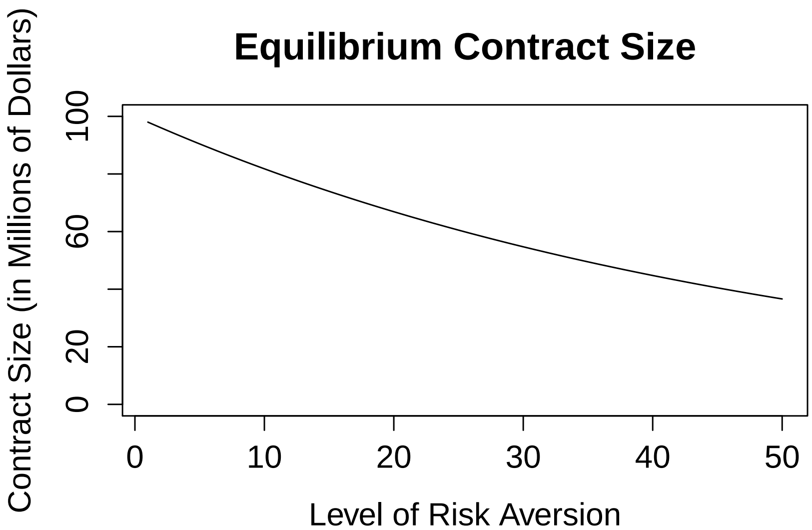

Figure 4: The team's payoff as a function of p for a given set of parameters. If player safety is low, the team achieves a greater payoff as safety increases. However, once safety is sufficiently high, the team fares worse in contract negotiations as safety further increases.

Without in-season contract negotiations, the team's optimal bargaining strategy follows one of two paths. First, it could offer greater amounts up front, enticing more types to accept before the season begins. However, since the team pays the contract, this greater amount hurts its overall payoff. Second, it could wait until the season ends. But this prevents the team from utilizing the player's level of risk aversion to its advantage, which once again decreases its payoff.

It is worth noting that the more risk averse types of the player benefit from forcing the team to increase its initial offer size. Thus, terminating negotiations during the season may appear to be a ploy from the player to increase his bargaining leverage. But the player cannot credibly commit to not receive offers during the season; when the team actually makes such an offer, it is in the player's own best interest to consider the team's proposition. As a result, threats to ignore such offers are incredible.

Nevertheless, there are instances in which terminating negotiations in-season is rational. Recall back to the complete information model, which showed that the player does not re-sign if $V < p^{2(\alpha-1)}n$. In such a scenario, contract spaces between the player and team do not exist; the player and team will not come to an agreement under any circumstances. Thus, the same result–free agency–is inevitable whether the parties negotiate during the season.

4 Conclusion

This article explored how risk aversion impacts contract extensions when temporary bargaining failure today could lead to the unexpected end of negotiations tomorrow. In the early parts of the season, the player's risk aversion opens up a larger contract space. However, the team can exploit this vulnerability by offering smaller amounts than if the player were less risk averse. As long as the player's outside option on the free agent market is sufficiently small, the team can re-sign the player. In some instances, the team can even re-sign the player during the preseason even though it will not be able to if the player opts for free agency.

That said, risk aversion is a difficult trait to measure. As a result, the team might not have a clear perception of the player's tolerance for risk. The second model in this article explored such a dynamic. Under the proper conditions, the team increases its offers as the season progresses. More risk averse types accept at the beginning of the process; less risk averse types reject, play games to credibly prove their tolerance for risk, and accept larger offers later on. Thus, the model provides a causal mechanism for why we observe players sign at various times during the season even though temporary bargaining failure does not destroy any of the pie as in standard bargaining models.

These results lead to two currently unanswered questions. With complete information, contract extensions exhibit a no-delay property; if the player will re-sign with the team, he does so immediately. Teams therefore benefit greatly by correctly identifying a player's level of risk aversion. While risk aversion as a type of preference is hard to observe, sports leagues provide public data on accepted contract offers and player characteristics. Given the causal mechanism described in this article, we could exploit these data to find possible correlations between observable characteristics to risk aversion.

Second, the risk of injury varies from sport to sport. We could utilize these differences to empirically investigate the effects of risk aversion on the negotiation process. Specifically, sports with lower levels of safety should see fewer players reach free agency *ceteris paribus*. However, an empirical study would need to be careful to control for league rules regarding free agency and possible selection bias.

5 Appendix

This appendix finishes the proof for Proposition 2. Specifically, we must prove step 6, which means we must show that it is optimal for the team to make an offer during the preseason that only the most risk averse type would accept.

To begin, note that if a type of player is willing to accept a contract in the preseason, all types more risk averse *must* accept that contract. To see this, consider an arbitrary risk aversion level α'. If that type is willing to accept an offer x from a generic period, it must be that the type's expected utility is greater for accepting than or equal to his greatest expected utility for accepting a later offer. Let r be the probability of reaching that offer and y be the value of that offer. Then for that type to be willing to accept the offer, it must be that:

$$x^{\frac{1}{\alpha'}} \geq r y^{\frac{1}{\alpha'}}$$
$$x \geq r^{\alpha'} y$$

Now consider an arbitrary type α'' that is more risk averse; that is, $\alpha'' > \alpha'$. Then to be willing to accept that same offer x, it must be that:

$$x^{\frac{1}{\alpha''}} \geq r y^{\frac{1}{\alpha''}}$$
$$x \geq r^{\alpha''} y$$

However, since $r \in (0, 1)$, we have $x \geq r^{\alpha'} y > r^{\alpha''} y$. Therefore, the more risk averse type receives a strictly greater expected utility from accepting x then advancing. As such, if a less risk averse type is willing to accept an offer, a more risk averse type *must* accept it.

Given that, the team can face four different situations at midseason: all three types remain unsigned, just the least risk averse and middle risk averse types remain unsigned, just the least risk averse type remans unsigned, and all types remain unsigned. Solving for the team's optimal offer and associated payoff for each of these outcomes determines the team's optimal strategy in the preseason.

First, suppose just the least risk averse type remains unsigned. That type accepts $x_m \geq p^{\alpha_L} n$ and rejects otherwise. Since the team's payoff is strictly decreasing in x_p, its optimal acceptable offer is $x_m = p^{\alpha_L} n$. The team earns $pV - p^{\alpha_L} n$ if it makes this offer and $p(V - n)$ if it induces the player to reject.[11] In turn, the team prefers making the optimal acceptable offer if:

$$pV - p^{\alpha_L} n > p(V - n)$$
$$p > p^{\alpha_L}$$

This holds. Thus, in terms of preseason payoffs, the team earns $pV - p^{\alpha_L} n$ for this outcome.

To induce the most risk averse and middle risk averse types to accept in the preseason, the team must offer x_p such that the middle risk averse type accepts. The middle risk averse type could accept and earn $x_p^{\frac{1}{\alpha_M}}$ or reject the offer in the preseason and accept $x_m = p^{\alpha_L} n$ at midseason, which gives him an expected payoff of $p(p^{\alpha_L} n)^{\frac{1}{\alpha_M}}$ from the perspective of the preseason. Therefore, he accepts x_p if:

$$x_p^{\frac{1}{\alpha_M}} \geq p(p^{\alpha_L} n)^{\frac{1}{\alpha_M}}$$
$$x_p \geq p^{\alpha_L + \alpha_M} n$$

The team's payoff is strictly decreasing in x_p. As such, its optimal acceptable offer which induces the two most risk averse types to accept in the preseason is $x_p = p^{\alpha_L + \alpha_M} n$. The team payoff is:

$$\frac{q_L}{q_L + q_M + q_H}(p^2 V - p^{1+\alpha_L} n) + \frac{q_M + q_H}{q_L + q_M + q_H}(p^2 V - p^{\alpha_L + \alpha_M} n)$$

$$p^2 V - n\left[\frac{q_L}{q_L + q_M + q_H} p^{1+\alpha_L} + \frac{q_M + q_H}{q_L + q_M + q_H} p^{\alpha_L + \alpha_M}\right]$$

In the second option, only the two types with the lowest levels of risk aversion advance to midseason. The fifth step showed that the team optimally offers $x_m = p^{\alpha_M} n$, the contract size that only the middle level risk averse type accepts, if $q_L < q_M \frac{p^{\alpha_L} - p^{\alpha_M}}{p - p^{\alpha_L}}$. The parameters of Proposition 2 assures this.

Note that the most risk averse type accepts in the preseason if accepting x_p generates a higher payoff than waiting to accept $x_m = p^{\alpha_M} n$, or:

$$x_p^{\frac{1}{\alpha_H}} \geq p(p^{\alpha_M} n)^{\frac{1}{\alpha_H}}$$
$$x_p \geq p^{\alpha_M + \alpha_H} n$$

The team's payoff is strictly decreasing in x_p if any type accepts with positive probability. Thus, its optimal acceptable offer that only the type with the highest level of risk aversion would accept is $x_p = p^{\alpha_M + \alpha_H} n$.

[11] The proof for step 1 shows that the team always offers $x_o = n$ in the offseason, which the player accepts.

Overall, the team earns the following payoff in expectation for these strategies:

$$\frac{q_L}{q_L+q_M+q_H}p^2(V-n)+\frac{q_M}{q_L+q_M+q_H}p(pV-p^{\alpha_M}n)+\frac{q_H}{q_L+q_M+q_H}p^2V-p^{\alpha_M+\alpha_H}n$$

$$p^2V-n\left[\frac{q_L}{q_L+q_M+q_H}p^2+\frac{q_M}{q_L+q_M+q_H}p^{1+\alpha_M}+\frac{q_H}{q_L+q_M+q_H}p^{\alpha_M+\alpha_H}\right]$$

The team prefers having the middle and least risk averse types advancing to midseason over having only the least risk averse type advancing to midseason if:

$$p^2V-n\left[\frac{q_L}{q_L+q_M+q_H}p^{1+\alpha_L}+\frac{q_M+q_H}{q_L+q_M+q_H}p^{\alpha_L+\alpha_M}\right]$$

$$>p^2V-n\left[\frac{q_L}{q_L+q_M+q_H}p^2+\frac{q_M}{q_L+q_M+q_H}p^{1+\alpha_M}+\frac{q_H}{q_L+q_M+q_H}p^{\alpha_M+\alpha_H}\right]$$

$$(q_L)p^2+(q_M)p^{1+\alpha_M}+(q_H)p^{\alpha_M+\alpha_H}>(q_L)p^{1+\alpha_L}+(q_M+q_H)p^{\alpha_L+\alpha_M}$$

$$q_L>\frac{q_M(p^{\alpha_L+\alpha_M}-p^{1+\alpha_M})+q_H(p^{\alpha_L+\alpha_M}-p^{\alpha_M+\alpha_H})}{p^2-p^{1+\alpha_L}}$$

Proposition 2 assures that this inequality holds. Therefore, the team cannot optimally structure its offers such that only the least risk averse type advances to midseason.

Third, suppose no type advances to midseason. Then, off the equilibrium path, the team must believe with probability 1 that the least risk averse type rejected x_p due to the D1 refinement. Therefore, at midseason, the team offers $x_m = p^{\alpha_L}n$, and the player accepts. In turn, to induce all types to accept during the preseason, the team optimally offers $x_p = p^{2\alpha_L}n$. The team earns $p^2V - p^{2\alpha_L}n$ for this outcome. This is worse than screening each type out if:

$$p^2V-n\left[\frac{q_L}{q_L+q_M+q_H}p^2+\frac{q_M}{q_L+q_M+q_H}p^{1+\alpha_M}+\frac{q_H}{q_L+q_M+q_H}p^{\alpha_M+\alpha_H}\right]$$

$$>p^2V-p^{2\alpha_L}n$$

$$p^{2\alpha_L}>\frac{q_L}{q_L+q_M+q_H}p^2+\frac{q_M}{q_L+q_M+q_H}p^{1+\alpha_M}+\frac{q_H}{q_L+q_M+q_H}p^{\alpha_M+\alpha_H}$$

$$q_L>\frac{q_M(p^{2\alpha_L-p^{1+\alpha_M}})+q_H(p^{2\alpha_L}-p^{\alpha_M+\alpha_H})}{p^2-p^{2\alpha_L}}$$

Again, Proposition 2 assures these parameters do not hold. As such, the team cannot optimally induce all types to accept the first offer.

Finally, the team could induce all types to advance to midseason. The remaining subgame is simply a two-period version of the bargaining game. At midseason, cannot optimally offer $x_m < p^{\alpha_H}n$. If it did, all types of players reject, and the team earns $p(V-n)$ from midseason perspective. Alternatively, if the team offers $x_m = p^{\alpha_H}n$, then only the high type accepts at midseason, the rest accept during the offseason, and the team earns $\frac{q_L}{q_L+q_M+q_H}pV - p^{\alpha_H}n + \frac{q_M+q_H}{q_L+q_M+q_H}p(V-n) > p(V-n)$. Thus, the team's optimal midseason contract offer cannot be less than $x_m = p^{\alpha_H}n$.

From there, note that only the team's optimal x_m must be $p^{\alpha_H}n$, $p^{\alpha_M}n$, or $p^{\alpha_L}n$. Any $x_m \in (p^{\alpha_H}n, p^{\alpha_M}n)$ induces just the high type to accept but provides a needless concession that $p^{\alpha_H}n$ does not. Likewise, any $x_m \in (p^{\alpha_M}n, p^{\alpha_L}n)$ induces the high and middle types to accept but provides them with a needless concession that $p^{\alpha_M}n$ does not. Lastly, any $x_m > p^{\alpha_L}n$ induces all types to accept but provides them with a needless concession that $p^{\alpha_L}n$ does not.

If $x_m = p^{\alpha_L}n$ is the optimal offer, the team could profitably deviate by offering $x_p = p^{2\alpha_L}n$; the team earns $p^2V - p^{2\alpha_L}n$ for the deviation but only $p^2V - p^{1+\alpha_L}n$ for maintaining the strategy. Thus, the team cannot optimally induce all types to reject in the preseason but then offer $x_m = p^{\alpha_L}n$ at midseason.

If $x_m = p^{\alpha_M}n$ is the optimal offer, the two most risk averse types accept, and the the least risk averse type accepts during the offseason. Under these conditions, the team earns:

$$p^2V - \frac{q_H + q_M}{q_H + q_M + q_L}p^{\alpha_M+1}n - \frac{q_L}{q_H + q_M + q_L}p^2n$$

Alternatively, the team could offer $x_p = p^{\alpha_H+\alpha_M}n$ and $x_m = p^{\alpha_M}n$. This induces the most risk averse type to accept during the preseason, the middle risk averse type to accept at midseason, and the least risk averse type to accept during the offseason. By playing this deviation, the team shifts its payoff to:

$$p^2V - \frac{q_H}{q_H + q_M + q_L}p^{\alpha_H+\alpha_M}n - \frac{q_M}{q_H + q_M + q_L}p^{\alpha_M+1}n - \frac{q_L}{q_H + q_M + q_L}p^2n$$

The only difference in these payoffs is when the team faces the most risk averse type. Thus, the deviation is profitable if:

$$p^{\alpha_M+1}n > p^{\alpha_H+\alpha_M}n$$

This holds. As such, the team cannot optimally induce all types to reject in the preseason but then offer $x_m = p^{\alpha_M}n$ at midseason.

If $x_m = p^{\alpha_H}n$ is the optimal offer, only the most risk averse type accepts at midseason, and the rest accept during the offseason. Under these conditions, the team earns:

$$p^2V - \frac{q_H}{q_H + q_M + q_L}p^{\alpha_H+1}n - \frac{q_M + q_L}{q_H + q_M + q_L}p^2n$$

Alternatively, the team could profitably deviate by offering $x_p = p^{\alpha_H+\alpha_L}$ and $x_m = p^{\alpha_L}n$. Under this deviation, only the most risk averse type accepts during the preseason and the rest accept at midseason. This time, the team earns:

$$p^2V - \frac{q_H}{q_H + q_M + q_L} p^{\alpha_H + \alpha_L} n - \frac{q_M + q_L}{q_H + q_M + q_L} p^{\alpha_L + 1} n$$

Consequently, this is a profitable deviation if:

$$p^2V - \frac{q_H}{q_H + q_M + q_L} p^{\alpha_H + \alpha_L} n - \frac{q_M + q_L}{q_H + q_M + q_L} p^{\alpha_L + 1} n$$

$$> p^2V - \frac{q_H}{q_H + q_M + q_L} p^{\alpha_H + 1} n - \frac{q_M + q_L}{q_H + q_M + q_L} p^2 n$$

$$q_H p^{\alpha_H + 1} + (q_M + q_L) p^2 > q_H p^{\alpha_H + \alpha_L} + (q_M + q_L) p^{\alpha_L + 1}$$

Note that $q_H p^{\alpha_H + 1} > q_H p^{\alpha_H + \alpha_L}$ and $(q_M + q_L)p^2 > (q_M + q_L)p^{\alpha_L + 1}$. Thus, the inequality holds, and the team cannot optimally induce all types to reject in the preseason but then offer $x_m = p^{\alpha_H} n$ at midseason.

For a numerical example, suppose $q_L = q_M = q_H = \frac{1}{3}$, $p = .99$, $\alpha_L = 5$, $\alpha_M = 7$, and $\alpha_H = 12$. Then, in equilibrium, the team offers approximately \$82.6 million during the preseason, and only the most risk averse type accepts. At midseason, the team offers approximately \$93.2 million, and only the middle risk averse type accepts. Finally, during the offseason, the team offers \$100 million, and the least risk averse type accepts.

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